Rules for the Rulemakers: Asymmetric Information and the Political Economy of Benefit-Cost Analysis—Online Appendix

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January 5, 2024

1 Introduction

This appendix contains seven sections. Section 2 proves that a cheap talk equilibrium in our model does not exist. Section 2 characterizes the equilibrium when we relax Assumptions 2 and 3 in the paper and allow for a positive direct cost for conducting a BCA. Section 4 presents and analyze a continuous-type version of our model. Section 5 presents an alternative model of BCA with bias. Section 6 presents the full commitment solution. Section 7 contains all proofs.

2 Does a cheap talk equilibrium exist?

In the model presented in the paper, we assume that the regulator cannot communicate with the executive about the benefit of the proposed rule. We now consider relaxing this assumption by allowing the regulator to decide on a messaging strategy $\mu : \mathcal{B} \to \Delta(\mathcal{M})$, where $\mathcal{B}$ is the set of possible benefits and $\mathcal{M}$ is an arbitrary message space. The executive can now condition its approval rule $\phi(\mu, \cdot)$ on the message it receives, in addition to any other information the executive has access to (such as the results of a BCA). As it turns out, the following lemma, which applies to all subgames in our analysis, states that this message must be uninformative; there is no equilibrium in any subgame in which the executive changes its acceptance decision on a message by the regulator. That is, we do not have a "cheap talk" equilibrium in the sense of Crawford and Sobel (1982).

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Lemma OA1 *There does not exist an equilibrium of the regulatory proposal subgame in which the regulator proposes regulations of types B' and B'' with messages $\mu' = \mu(B')$ and $\mu'' = \mu(B'')$ such that the executive's equilibrium approval probability $\phi' = \phi(\mu', \cdot) \neq \phi(\mu'', \cdot) = \phi''$.*

The intuition is straightforward: as long as $\beta B > C$ (which is a necessary condition for the regulator to propose a new rule), the regulator's expected benefit increases in the approval probability. Thus, conditional on proposing, the regulator will send whatever message it needs to send to ensure that the probability of approval is as high as possible. As far as the executive is concerned, then, there is nothing meaningful to infer from messages sent by the regulator about the rule's social benefits.

3 Relaxing assumptions in the two-type model

In this paper, we characterized the equilibrium in our two-type model under Assumption 2 ($\beta > \frac{C}{B^L} + \frac{\frac{k_r}{1-q}}{B^L}$), and later in the paper, we stated (but did not prove) the equilibrium under a BCA mandate when we replaced Assumption 2 with Assumption 3 ($\beta \in \left(\frac{C+k_r}{B^L}, \frac{C}{B^L} + \frac{\frac{k_r}{1-q}}{B^L}\right)$). In this section, we characterize the equilibrium for all values of β. In addition, we allow for the BCA to have either a zero or a positive cost borne by both the executive and the regulator. We denote that cost k_a, and we let $k = k_r + k_a$. Since $k_a \geq 0$, $k \geq k_r$.

3.1 Prohibited BCA

Let us begin with the case of a BCA prohibition. If $\beta < \frac{C+k_r}{B^H}$, neither type of regulator would benefit from proposing, even if the executive were to approve the new rule with certainty.[1] In this case, it is straightforward to show that there is a unique equilibrium with $\rho_n^* = (0, 0)$ and $\phi_n^* = 1$.[2]

If $\beta \in \left(\frac{C+k_r}{B^H}, \frac{C+k_r}{B^L}\right)$, we have the following.

Proposition OA1 *If Assumption 1 in the paper holds and $\beta \in \left(\frac{C+k_r}{B^H}, \frac{C+k_r}{B^L}\right)$, then when BCA is prohibited, the unique equilibrium is $\rho_n^* = (1, 0)$ and $\phi_n^* = 1$.*

This is a case in which the executive and the regulator are aligned in that the regulator proposes in precisely those circumstances in which the executive wants it to propose. There is essentially no agency problem between the executive and the regulator, and it is as if there is symmetric information between the two parties.

[1] When $\beta = \frac{C+k_r}{B^H}$ there are infinitely many equilibrium in which a high-benefit regulator randomizes between proposing and not proposing, a low-benefit regulator does not propose, and the regulator approves a proposal with probability one. In the remainder of this section, we ignore knife-edge cases such as this.

[2] As in the paper, we assume employ the D1 refinement throughout this section. D1 implies that $\phi_n^* = 1$ is the unique approval probability.

If $\beta > \frac{C+k_r}{B^L}$ (which is an implication of Assumption 2 in the paper), then Proposition 1 from the paper holds. Summing up this analysis, then, as we move from $\beta < \frac{C+k_r}{B^H}$ to $\beta \in \left(\frac{C+k_r}{B^H}, \frac{C+k_r}{B^L}\right)$ to $\beta > \frac{C+k_r}{B^L}$, we move from a situation in which the regulator does not propose under any circumstances to one in which only a high-benefit regulator proposes (and thus proposes in exactly the circumstances the executive wants the regulator to propose) to one in which both types of the regulator propose (with a high-benefit regulator proposing with certainty and a low-benefit regulator proposing with a probability less than one if the executive is regulation averse ($\alpha < \overline{\alpha}_n$) and probability one if the executive is regulation sympathetic ($\alpha > \overline{\alpha}_n$).

3.2 Mandated BCA

Analogous to the BCA-prohibited case, if $\beta < \frac{C+k}{B^H}$, neither type of regulator would benefit from proposing, even if the executive were to approve the new rule with certainty.

If $\beta \in \left(\frac{C+k}{B^H}, \frac{C+k}{B^L}\right)$, we can establish

Proposition OA2 *If Assumption 1 in the paper holds and* $\beta \in \left(\frac{C+k}{B^H}, \frac{C+k}{B^L}\right)$, *the unique equilibrium is* $\rho_m^* = (1,0)$ *and* $\phi_m^* = (1,1)$.

In this case, the set of regulations the executive wants to approve are the same set. Therefore, under no circumstances will the regulator propose a regulation that the executive does not want to accept, so the signal given by BCA does not change the executive's certainty that it wants to approve all proposed regulations.

Next, we prove a more general version of Proposition 2 in the paper that allows for either Assumption 2 or Assumption 3 to hold. In the statement of the proposition, $\overline{\beta}_m \equiv \frac{C}{B^L} + \frac{\frac{k_r}{1-q}}{B^L}$, so whether Assumption 2 or Assumption 3 holds depends on whether $\beta > \overline{\beta}_m$ or $\beta \in \left(\frac{C+k}{B^L}, \overline{\beta}_m\right)$.

Proposition OA3 *If Assumption 1 in the paper holds and* $\beta > \frac{C+k}{B^L}$, *There exists an equilibrium in the regulatory proposal subgame. Assuming that equality conditions between parameters are not satisfied, this equilibrium is unique.*[3]

(1) If $\alpha \geq \overline{\alpha}_m(b^L) = \frac{C}{E[\widetilde{B}|\widetilde{b}=b^L]} = \frac{[p(1-q)+(1-p)q]C}{p(1-q)B^H+(1-p)qB^L}$, *then the equilibrium is* $\rho_m^* = (1,1)$, $\phi_m^* = (1,1)$.

(2) If $\alpha \leq \overline{\alpha}_m(b^L)$ *and* $\beta \leq \overline{\beta}_m$, *then the equilibrium is* $\rho_m^* = (1, \rho_m^*(B^L))$, $\phi_m^* = (1, \phi_m^*(b^L))$, *where*

$$\rho_m^*(B^L) = \frac{p(1-q)(\alpha B^H - C)}{(1-p)q(C - \alpha B^L)} \in (0,1],$$
$$\phi_m^*(b^L) = \frac{k}{q(\beta B^L - C)} - \frac{1-q}{q} \in (0,1].$$

[3] If any of the below weak inequalities are satisfied with equality, we get multiple equilibria.

(3) If $\alpha \in [\overline{\alpha}_m(b^H), \overline{\alpha}_m(b^L)]$ *and* $\beta \geq \overline{\beta}_m$*, where* $\overline{\alpha}_m(b^H) = \frac{C}{E[\widetilde{B}|\widetilde{b}=b^H]} = \frac{[pq+(1-p)(1-q)]C}{pqB^H+(1-p)(1-q)B^L} < \overline{\alpha}_m(b^H)$*, then the equilibrium is* $\rho^*_m = (1,1)$*,* $\phi^*_m = (1,0)$*.*

(4) If $\alpha \leq \overline{\alpha}_m(b^H)$ *and* $\beta \geq \overline{\beta}_m$*, then the equilibrium is* $\rho^*_m = (1, \rho^*_m(B^L))$*,* $\phi^*_m = (\phi^*_m(b^H), 0)$*, with*

$$\rho^*_m(B^L) = \frac{pq(\alpha B^H - C)}{(1-p)(1-q)(C-\alpha B^L)} \in (0,1],$$

$$\phi^*_m(b^H) = \frac{k}{(1-q)(\beta B^L - C)} \in (0,1].$$

Cases 1, 3, and 4 correspond to the Proposition 2 in the paper. Case 2 corresponds to the characterization of the equilibrium when Assumption 3 holds. The proof of Proposition OA3 in the appendix to this document is identical to the proof of Proposition 2 in the paper, but it also presents the proof of the equilibrium in case 2, which is not presented in the paper.

3.3 Comparing mandated BCA to prohibited BCA

When Assumption 1 in the paper holds and $\beta > \frac{C+k}{B^H}$ and $k_a > 0$, the expression for $\Delta^E(\alpha, \beta) = EU^E_m - EU^E_n$ involves nine cases instead of the four that arise when Assumption 2 holds. (We write $\Delta^E(\cdot)$ as dependent on β as well as α to emphasize that the gain or loss to the executive from using BCA depends on the regulator's welfare weight as well as the executive's.) These cases are summarized in Figure OA1.[4] Table OA1 shows the expressions for $\Delta^E(\alpha, \beta)$, while Table OA2 presents the decomposition terms for each of the nine cases. In Table OA2, the phrase "severe misalignment" refers to $\beta > \overline{\beta}_m$ (the case analyzed in the paper); "modest misalignment" refers to $\beta \in \left(\frac{C+k}{B^L}, \overline{\beta}_m\right)$ (Assumption 3 in the paper); " BCA alignment" refers to $\beta \in \left(\frac{C+k_r}{B^L}, \frac{C+k}{B^L}\right)$; and "full alignment" refers to $\beta \in \left(\frac{C+k}{B^H}, \frac{C+k_r}{B^L}\right)$. In the BCA alignment case, a positive direct cost of a BCA makes it welfare-reducing for a low-benefit regulator to propose with a BCA whereas a low-benefit regulator would be willing to propose with a high enough approval probability when BCA is not used.

3.4 Summing up

Expanding the range of β beyond that implied by Assumption 2 results in three changes in the analysis. First, it opens up the possibility that the regulator's interests are aligned with the executive's when BCA is mandated or prohibited, i.e., a low-benefit regulator would not propose but a high-benefit regulator would. In this case, the unique equilibrium with or without a BCA mandate entails the regulator behaving exactly

[4] The figure assumes that $\frac{C+k}{B^H} < \frac{C+k_r}{B^L}$, which holds provided (as seems natural) k_a is small in comparison to k_r. When $\frac{C+k}{B^H} \geq \frac{C+k_r}{B^L}$, cases 7 and 8 no longer arise and case 9 holds for all $\beta \in \left(\frac{C+k}{B^H}, \frac{C+k}{B^L}\right)$.

Case	α	β	$\Delta^E(\alpha,\beta)$
1	$\alpha \in (\overline{\alpha}_m(b^L), \frac{C}{B^L})$	$\beta > \frac{C+k}{B^L}$	$-k_a$
2	$\alpha \in \left[\overline{\alpha}_n, \overline{\alpha}_m(b^L)\right]$	$\beta > \overline{\beta}_m$	$(1-p)q(C-\alpha B^L) - p(1-q)(\alpha B^H - C) - k_a$
3	$\alpha \in \left[\overline{\alpha}_m(b^H), \overline{\alpha}_n\right]$	$\beta > \overline{\beta}_m$	$pq(\alpha B^H - C) - (1-p)(1-q)(C-\alpha B^L) + (1-p)\left[\frac{p(\alpha B^H - C)}{(1-p)(C-\alpha B^L)} - 1\right] k_r - k_a$
4	$\alpha \in \left(\frac{C}{B_H}, \overline{\alpha}_m(b^H)\right]$	$\beta > \overline{\beta}_m$	$-p\left(\frac{2q-1}{q}\right)\left(\frac{\alpha B^H - C}{C-\alpha B^L}\right) k_r - p\left[1 + \frac{q}{1-q}\left(\frac{\alpha B^H - C}{C-\alpha B^L}\right)\right] k_a$
5	$\alpha \in \left(\frac{C}{B^H}, \overline{\alpha}_n\right]$	$\beta \in (\frac{C+k}{B^L}, \overline{\beta}_m)$	$p\left(\frac{2q-1}{q}\right)\left(\frac{\alpha B^H - C}{C-\alpha B^L}\right)\left[C - \alpha B^L + k_r\right] - p\left[1 + \frac{1-q}{q}\left(\frac{\alpha B^H - C}{C-\alpha B^L}\right)\right] k_a$
6	$\alpha \in \left[\overline{\alpha}_n, \overline{\alpha}_m(b^L))\right]$	$\beta \in (\frac{C+k}{B^L}, \overline{\beta}_m)$	$(1-p)\left[1 - \frac{1-q}{q}\frac{p}{1-p}\left(\frac{\alpha B^H - C}{C-\alpha B^L}\right)\right]\left[C - \alpha B^L + k_r\right] - p\left[1 + \frac{1-q}{q}\left(\frac{\alpha B^H - C}{C-\alpha B^L}\right)\right] k_a$
7	$\alpha \in \left(\frac{C}{B_H}, \overline{\alpha}_n\right]$	$\beta \in (\frac{C+k_r}{B^L} \frac{C+k}{B^L})$	$p\left(\frac{\alpha B^H - C}{C-\alpha B^L}\right)\left[C - \alpha B^L + k_r\right] - pk_a$
8	$\alpha \in \left[\alpha_n, \frac{C}{B^L}\right)$	$\beta \in (\frac{C+k_r}{B^L} \frac{C+k}{B^L})$	$(1-p)(C - \alpha B^L + k_r) - pk_a$
9	$\alpha \in (\frac{C}{B_H}, \frac{C}{B^L})$	$\beta \in (\frac{C+k}{B^H}, \frac{C+k}{B^L})$	$-pk_a$

Table OA1: Expression for $\Delta^E(\alpha,\beta)$ as a function of α and β.

Case	**Executive's regulatory preference?**	**Alignment?**	Δ_1	Δ_2	Δ_3	**Selection?**
1	Regulation sympathetic	Modest or severe misalignment	0	0	0	BCA does not change selection
2	Regulation sympathetic	Severe misalignment	$-$	$+$	0	BCA does not change selection
3	Regulation averse	Severe misalignment	$+$	$-$	$-$	BCA worsens selection
4	Regulation averse	Severe misalignment	$+$	$-$	$-$	BCA worsens selection
5	Regulation averse	Modest misalignment	$+$	?*	$+$	BCA improves selection
6	Regulation sympathetic	Modest misalignment	$-$	$+$	$+$	BCA improves selection
7	Regulation averse	BCA alignment	$+$	$+$	$+$	BCA improves selection
8	Regulation sympathetic	BCA alignment	0	$+$	$+$	BCA improves selection
9	Regulation averse or sympathetic	Full alignment	0	0	0	BCA does not change selection

*$+$ if $k_a = 0$

Table OA2: Signs of Δ_1, Δ_2, and Δ_3.

as the executive most prefers. In this case, (case 9 in Figure OA1 and Tables OA1 and OA2) BCA adds nothing except possibly for a direct cost, as can be seen in Table OA1. This case of full alignment is useful as a theoretical benchmark, but its empirical relevance seems questionable. The literature (cited in the paper) emphasizing the role of executive branch gatekeepers (like OIRA) and BCA in resolving principal-agent problems between regulatory agencies and the executive administration suggests that full alignment is unlikely to be the norm.

Second, extending the range of β creates the possibility that the regulator would propose in the manner the executive most prefers under a BCA mandate, but does not do so without BCA. In these cases (cases 7 and 8 in Tables OA1 and OA2) BCA can potentially be valuable to the regulator, not because of its informational content, but because its direct cost k_a works to screen the types of the regulator. This case is a theoretical curiosity, but as with the case of full alignment, not empirically relevant. The idea that the cost of a BCA (which is likely to be small as a compared to other proposal costs) operates by itself to discipline the behavior of regulatory agencies, is dubious.

Third, extending the range of β to that specified in Assumption 3 in the paper ($\beta \in \left(\frac{C+k}{B^L}, \overline{\beta}_m\right)$) opens up the possibility of a less severe agency problem than the one featured in the paper. This assumption has some bite. As discussed in the paper, and as can be seen from Table OA1 for the case of $k_a = 0$, $\Delta^E(\alpha, \beta) > 0$ for all $\alpha \in \left(\frac{C}{B^H}, \frac{C}{B^L}\right)$. Thus, as long as its direct cost is sufficiently small, the executive benefits from a BCA mandate. Thus, while a BCA mandate can harm the executive when the agency problem between the regulator and the executive is sufficiently severe in the sense of Assumption 2 (even if BCA is costless), this is not the case when the agency problem is somewhat less severe.

Despite this point, we feature the case of $\beta > \overline{\beta}_m$ in the paper because we believe it likelier to obtain in practice than the case of $\beta \in (\frac{C+k}{B^L}, \overline{\beta}_m)$. The length of the range is $\overline{\beta}_m - \frac{C+k}{B^L} = \frac{qk}{(1-q)B^L}$. When q is bounded away from one, and when k is small relative to B^L, this range will be small. As we argue in Section 4.4 when we discuss parameterizing the continuous-type version of our model, both are likely to be the case. To put numbers to the discussion, if $B^L = 200, C = 150, B^H = 700, k_r = 4$, and $k_a = 0.002$, (which are numbers similar to the baseline parameterization of the continuous-type model) then for a reasonably informative BCA signal of $q = 0.75$, $\frac{C+k}{B^L} = 0.77$ and $\overline{\beta}_m = 0.83$. As such, the interval of "modest misalignment" is small. Thus, while the cases embodied by Assumption 3 yields interesting and important differences relative to those implied by Assumption 2, they occupy a relatively small portion of parameter space unless the BCA is extremely informative.

4 Continuous-type model

This section presents a continuous-type version of our model. We maintain the assumption that there is direct cost k_a of a BCA and, as in the previous section, we let $k = k_r + k_a \geq k_r$.

Throughout this section we distinguish between, on the one hand, lemmas and propositions that are established through formal arguments and, on the other hand, results. A result either establishes a possibility through a numerical example or summarizes a regularity through a systematic exploration of the parameter space.

We assume the executive's prior is that B is a normally distributed random variable $\widetilde{B} \sim N(B_0, \sigma_0)$, with associated density, distribution, and reverse distribution functions $f_0(B) = \frac{1}{\sigma_0}\psi\left(\frac{B-B_0}{\sigma_0}\right)$, $F_0(B) = \Psi\left(\frac{B-B_0}{\sigma_0}\right)$, and $\widehat{F}_0(B) = 1 - F_0(B) = \widehat{\Psi}\left(\frac{B-B_0}{\sigma_0}\right)$, where $\psi(\cdot)$, $\Psi(\cdot)$, and $\widehat{\Psi}(\cdot) = 1 - \Psi(\cdot)$ are the density, distribution, and reverse distribution functions of the standard normal distribution.

BCA reveals a possibly noisy signal $\widetilde{b} = B + \sigma_b \widetilde{\varepsilon}_b$ of B to the executive, where $\sigma_b \geq 0$ and $\widetilde{\varepsilon}_b \sim N(0,1)$. Given B, the density, distribution, and reverse distribution functions of $\widetilde{b}$ are $\frac{1}{\sigma_b}\psi\left(\frac{b-B}{\sigma_b}\right)$, $\Psi\left(\frac{b-B}{\sigma_b}\right)$, and $\widehat{\Psi}\left(\frac{b-B}{\sigma_b}\right)$ respectively.

An executive's *ex ante* posture toward regulation is reflected by $\widehat{F}_0\left(\frac{C}{\alpha}\right)$, which we define as the executive's *regulation-sympathy index*. This index is the executive's *ex ante* probability of facing a rule it would like to accept. Accordingly, we characterize an executive as *regulation neutral/averse/sympathetic* if $\widehat{F}_0\left(\frac{C}{\alpha}\right) = / < / > 0.5$ or, equivalently, if $\alpha = / < / > \frac{C}{B_0}$. Analogously, we let $\widehat{F}_0\left(\frac{C+k}{\beta}\right)$ serve as the regulator's regulation-sympathy index, and call the regulator regulation neutral/averse/sympathetic if $\widehat{F}_0\left(\frac{C+k}{\beta}\right) = / < / > 0.5$. Because the regulator does not share the executive's priors (since it knows B), this index can be interpreted as the executive's prior of the regulator's sympathy to new rules.

Symmetric information benchmark We derive specific results regarding the equilibrium outcomes for each subgame in the remainder of this section, but before that it is useful to present the symmetric information benchmark. If the executive knew B, it would approve a rule proposed by the regulator if $\alpha B - C - k_r \geq -k_r$, or equivalently $B \geq \frac{C}{\alpha}$. The executive's *approval function* is therefore $I\{B \geq \frac{C}{\alpha}\}$. Given this, the regulator proposes a new rule if $I\{B \geq \frac{C}{\alpha}\}[\beta B - C] - k_r \geq 0$. If $\alpha \geq \frac{C}{C+k_r}\beta$, the regulator proposes a rule if $B \geq \frac{C+k_r}{\beta}$, and the executive would accept it because $B \geq \frac{C+k_r}{\beta} \geq \frac{C}{\alpha}$. In this case, the regulator refrains from proposing rules with $B \in \left[\frac{C}{\alpha}, \frac{C+k_r}{\beta}\right)$ that the executive would have accepted because they would make the regulator worse off. If $\alpha < \frac{C}{C+k_r}\beta$, the regulator proposes a rule if $B \geq \frac{C}{\alpha}$, and the executive accepts it. In this case, the regulator refrains from proposing rules with $B \in \left[\frac{C+k_r}{\beta}, \frac{C}{\alpha}\right)$ that it would benefit from because it knows the executive will reject them. Viewed *ex ante*, the executive's expected welfare in the symmetric information benchmark is

$$EU_f^E = \int_{B_f^*}^{\infty} [\alpha x - C - k_r] f_0(x) dx, \tag{1}$$

where $B_f^* = \max\left\{\frac{C}{\alpha}, \frac{C+k_r}{\beta}\right\}$.

4.1 Regulatory proposal subgame: prohibited BCA

The only inference the executive can make about the regulator's private information B when the executive prohibits BCA comes from the event that the regulator proposes a rule in the first place. Suppose, then, the regulator conjectures that the executive's approval probability is $\phi_n \in (0, 1]$.[5] or

$$B \geq B_n(\phi_n) \equiv \frac{k_r}{\beta \phi_n} + \frac{C}{\beta}. \tag{2}$$

We refer to $B_n(\phi_n)$ as the *pivotal benefit.*

If the executive conjectures a pivotal benefit B_n, the executive updates its belief over B as $\widetilde{B}|\widetilde{B} \geq B_n$. The executive's acceptance probability $\phi_n(B_n)$ is then given by

$$\phi_n(B_n) = \begin{cases} 1 & \text{if } \alpha E[\widetilde{B}|\widetilde{B} \geq B_n] - C \geq 0 \\ \in [0,1] & \text{if } \alpha E[\widetilde{B}|\widetilde{B} \geq B_n] - C = 0 \\ 0 & \text{if } \alpha E[\widetilde{B}|\widetilde{B} \geq B_n] - C \leq 0 \end{cases}, \tag{3}$$

where $E[\widetilde{B}|\widetilde{B} \geq B_n] = B_0 + \sigma_0 h\left(\frac{B_n - B_0}{\sigma_0}\right)$, and $h(z) = \frac{\psi(z)}{\widehat{\Psi}(z)}$ is the hazard function for the standard normal distribution. We note that the structure of our model is what Tirole (2016) calls an *anti-lemons* environment in that a proposal by the regulator makes the executive's posterior expectation higher than its prior expectation. In this sense, the regulator plays a potentially valuable screening role for the executive.

An equilibrium is a strategy pair $\{B_n^*, \phi_n^*\}$ such that the regulator's proposal strategy and the executive's approval strategy are each other's best responses, i.e., $B_n^* = B_n(\phi_n^*)$ and $\phi_n^* = \phi_n(B_n^*)$. The next proposition formally characterizes the equilibrium.

Proposition OA4 *If BCA is prohibited in the regulatory process, an equilibrium in the regulatory proposal subgame,* $\{B_n^*, \phi_n^*\}$, *exists and is unique up to changes of measure zero.*[6] *In that equilibrium, the regulator proposes a new rule if* $B > B_n^*$ *and does not propose a new rule if* $B < B_n^*$. *The regulator accepts the proposed*

[5] As in the main text, we continue to use as our equilibrium concept (weak)perfect Bayesian equilibrium, equipped with the D1 refinement, ensuring that a proposal is on path and therefore the executive uses Bayes' rule to update its beliefs.

[6] That is, any two equilibria are identical modulo the regulator's behavior when $B = B_n^*$.

rule with probability ϕ_n^* *and rejects it with probability* $1-\phi_n^*$. *(1) For* $\alpha < \alpha_n^*(\beta) \equiv \frac{C}{B_0+\sigma_0 h\left(\frac{\frac{C+k_r}{\beta}-B_0}{\sigma_0}\right)} \in (0, \frac{C}{C+k_r}\beta)$*, the equilibrium is given by*

$$B_n^* = \frac{C}{\beta} + \frac{k_r}{\beta\phi_n^*} \in (\frac{C+k_r}{\beta}, \frac{C}{\alpha}) \tag{4}$$

$$\phi_n^* \in (0,1),$$

where ϕ_n^* *is the solution for* ϕ *to*

$$E\left[\widetilde{B}|\widetilde{B} \geq \frac{C}{\beta} + \frac{k_r}{\beta\phi}\right] - \frac{C}{\alpha} = 0. \tag{5}$$

(2) For $\alpha \geq \alpha_n^*(\beta)$*, the equilibrium is given by*

$$B_n^* = \frac{C+k_r}{\beta} \tag{6}$$

$$\phi_n^* = 1. \tag{7}$$

We note that case 1 in Proposition OA4 is analogous to case 1 in Proposition 1 in the paper which characterizes the equilibrium in the prohibited-BCA subgame in the two-type model. Case 2 in Proposition OA4 here encompasses case 2 in Proposition 1 of the two-type model.[7] In both models, an executive that places a sufficiently high weight on the benefits of the prospective rule will always approve a proposed regulation. In particular, Proposition OA4 implies that in the continuous-type model when the regulator's preferences are the same as the executive's, $\alpha = \beta$, the executive approves the proposed rule with certainty.

To understand why, in the continuous-type model, an executive with $\alpha < \alpha_n^*(\beta)$ might reject a proposal, suppose the regulator believed the executive would approve any proposed rule with certainty. The regulator would propose a rule if $B \geq \frac{C+k_r}{\beta}$, and the executive's expected net benefit from approving the regulation would be $\alpha E\left[\widetilde{B}|\widetilde{B} \geq \frac{C+k_r}{\beta}\right] - C$. We show in the proof of the proposition that when $\alpha < \alpha_n^*(\beta)$, $\alpha E\left[\widetilde{B}|\widetilde{B} \geq \frac{C+k_r}{\beta}\right] - C < 0$. By rejecting rules with positive probability, the executive can entice the regulator to forego proposing lower-quality rules, increasing the executive's expected payoff. Thus, the executive hurts its interests if it approves any rule proposed by the regulator with certainty.

The executive's gatekeeper power is limited by its lack of information, leading to three distinct inefficiencies relative to the symmetric information outcome, inefficiencies that are directly analogous to those in the two-type model in the paper: type 1 error from rejecting rules with $\alpha B - C > 0$, type 2 error from accepting rules with $\alpha B - C < 0$, and excess proposal cost from the regulator proposing more types of rules than in the

[7] Case 2 in Proposition OA4 also encompasses what we refer to in the previous section as the case of full alignment, a case that arose when we relaxed Assumption 2 in the paper.

symmetric information case. We can decompose the executive's deadweight loss—$DWL^E = EU_f^E - EU_n^E$—to illustrate these three sources of inefficiency:

$$\begin{aligned} DWL^E &= EU_f^E - \int_{B_n^*}^{\infty} \{\phi_n^* [\alpha x - C] - k_r\} f_0(x)dx \\ &= DWL_1^E + DWL_2^E + DWL_3^E, \end{aligned} \tag{8}$$

where

$$DWL_1^E = \int_{\frac{C}{\alpha}}^{\infty} \left(I\{x \geq B_f^*\} - \phi_n^* I\{x \geq B_n^*\}\right) [\alpha x - C] f_0(x)dx \tag{9}$$

$$DWL_2^E = \int_{-\infty}^{\frac{C}{\alpha}} \left(I\{x \geq B_n^*\}\phi_n^* - I\{x \geq B_f^*\}\right) [C - \alpha x] f_0(x)dx \tag{10}$$

$$DWL_3^E = k_r \left[F_0(B_f^*) - F_0\left(B_n^*\right)\right]. \tag{11}$$

DWL_1^E, DWL_2^E, and DWL_3^E are the executive's welfare losses due to type 1 errors, type 2 errors, and excess expected proposal costs respectively. These are directly analogous to components of the deadweight loss in the two-type model.

These deadweight loss components depend on α (holding all other parameters fixed). When $\alpha \geq \frac{C}{C+k_r}\beta$, Proposition OA4 implies that the equilibrium outcome coincides with the symmetric information benchmark, so there is no deadweight loss. If $\alpha \in \left[\alpha_n^*(\beta), \frac{C}{C+k_r}\beta\right)$, $DWL_1^E = 0$ but $DWL_i^E > 0, i = 2, 3$. From Proposition OA4 $\phi_n^* = 1$ and $\frac{C}{C+k_r}\beta = B_n^* < B_f^* = \frac{C}{\alpha}$ in this case, so the executive never rejects a proposal with $B > \frac{C}{\alpha}$, i.e., it will not make a type 1 error. But it would accept a proposal with $B \in (\frac{C+k_r}{\beta}, \frac{C}{\alpha})$, so the executive could make a type 2 error. And expected proposal costs are excessive because the regulator's equilibrium proposal range is greater than the symmetric information range. When $\alpha < \alpha_n^*(\beta)$, $DWL_i^E > 0, i = 1, 2, 3$. This is because $\phi_n^* < 1$, so a proposed rule with $B > \frac{C}{\alpha}$ could be rejected by the executive, a type 1 error. A type 2 error can also arise because the pivotal benefit $B_n^* < \frac{C}{\alpha}$ and the approval probability $\phi_n^* > 0$. And the expected proposal cost is excessive because $B_n^* < B_f^*$.

Figure OA2 shows the magnitudes of decomposition components for a particular parameterization with $\alpha = 0.25$ and other parameters at the baseline levels of our computational analysis. (For these baseline levels and a discussion thereof, see Table OA3 and the surrounding discussion in Section 4.4.) The no-BCA equilibrium in this case is $B_n^* = 434.75$ and $\phi_n^* = 0.014$. (The graphs bounding the areas for the type 1 and type 2 error are the integrands of DWL_1, DWL_2, and DWL_3.) The welfare loss due to a type 1 error, DWL_1, is substantial. With $\sigma_0 = 200$, there is a high likelihood that the executive rejects proposals that would it would have benefitted from. Because ϕ_n^* is quite close to zero, the welfare loss from a type 2 error,

DWL_2, is small. The welfare loss from excess proposal cost, DWL_3, is non-trivial because the executive's prior probability of a rule falling in the range proposed in the prohibited BCA subgame but not under symmetric information, $[B_n^*, B_f^*] = [434.76, 600]$, is sizable, as the mean of the executive's prior distribution B_0 is 450.

When $\alpha < \alpha_n^*(\beta)$, we have the following comparative statics: $\frac{\partial \phi_n^*}{\partial \alpha} > 0$, $\frac{\partial \phi_n^*}{\partial \beta} < 0$, $\frac{\partial B_n^*}{\partial \alpha} < 0$ and $\frac{\partial B_n^*}{\partial \beta} = 0$.[8] Thus, as α decreases *ceteris paribus*—i.e., as the executive becomes more regulation averse—the regulator proposes higher quality rules, but the likelihood the executive approves them decreases. Thus a more regulation-averse executive trades off a higher risk of type 1 error for a lower risk of type 2 error. This makes sense: a regulation-averse executive is hurt more by approving low-benefit rules than by failing to approve high-benefit rules. Further, as β increases, holding all else fixed, the agency problem between the executive and regulator worsens, and the executive similarly trades off a higher risk of type 1 error for a lower risk of type 2 error by lowering the probability of acceptance. This also makes sense: a more regulation-sympathetic regulator is inclined to propose rules that would hurt the executive, and the lower equilibrium approval probability disciplines the regulator to some extent, even if it means that the executive may reject some high-benefit proposals. Both of these effects have direct analogues in the two-type model that are demonstrated in the main text; the effect as α changes is discussed in Section 3.1, while the effect as β changes is demonstrated by moving between Assumptions 2 and 3.

4.2 Regulatory proposal subgame: mandatory BCA

We now turn to mandatory BCA. Throughout, we assume that $\sigma_b > 0$ or $k_a > 0$. That is, the BCA is noisy or costly or both.[9]

Suppose the regulator conjectures the executive's approval probability as a function of the measured benefit b is $\phi_m(b)$, and further suppose that $\phi_m(\cdot)$ is believed to be a non-decreasing function of b. As we show presently, this is indeed how an optimizing executive would behave, so an expectation of a non-decreasing approval function is confirmed in equilibrium.[10] The regulator's best response is to propose only

[8] The comparative statics for ϕ_n^* are established in the proof of Proposition OA4. The result $\frac{\partial B_n^*}{\partial \alpha} < 0$ follows from condition (4). From ((4), (5), and the expression for a lower-truncated normal expectation, we have $B_0 + \sigma_0 h\left(\frac{B_n^* - B_0}{\sigma_0}\right) = \frac{C}{\alpha}$, from which it directly follows that $\frac{\partial B_n^*}{\partial \beta} = 0$.

[9] The case of perfect and costless BCA ($\sigma_b = 0, k_a = 0$) is straightforward: it enables the executive to replicate the symmetric-information equilibrium, thus eliminating the deadweight loss (8). The executive could never be worse off mandating perfect and costless BCA. In particular, from the analysis in the previous section, when $\alpha \geq \frac{C}{C+k_r}\beta$, the regulator is neither better off nor worse off mandating perfect and costless BCA. When $\alpha < \frac{C}{C+k_r}\beta$, it is strictly better off.

[10] An expectation by the regulator that the executive will be more likely to approve proposals with higher measured benefits is also compelling for other reasons. It is consistent with an agency believing that executive will not behave arbitrarily in the face of hard evidence from a BCA. An expectation of a greater likelihood by the executive branch to approve proposals with higher measured benefits would also be natural if the regulator believed the executive branch was averse to court challenges of its decisions.

when $E_{\widetilde{b}}\left[\phi_m(\widetilde{b})|B\right][\beta B - C] - k \geq 0$.

Suppose that $E_{\widetilde{b}}\left[\phi_m(\widetilde{b})|B\right] > 0$.[11] This implies that for $B > \frac{C}{\beta}$ (which is necessary for the regulator to propose a new rule), $E_{\widetilde{b}}\left[\phi_m(\widetilde{b})|B\right][\beta B - C]$ is strictly increasing in B because a higher B shifts the distribution of $\widetilde{b}$ in the sense of first-order stochastic dominance, and thus the expectation $E_{\widetilde{b}}\left[\phi_m(\widetilde{b})|B\right]$ is non-decreasing in B. Further, $\lim_{B\to\infty} E_{\widetilde{b}}\left[\phi_m(\widetilde{b})|B\right][\beta B - C] - k = \infty$ and $\lim_{B\to\frac{C}{\beta}} E_{\widetilde{b}}\left[\phi_m(\widetilde{b})|B\right][\beta B - C] - k = -k < 0$. Thus, the intermediate value theorem implies that there exists a unique pivotal benefit $B_m > \frac{C}{\beta}$ such that $E_{\widetilde{b}}\left[\phi_m(\widetilde{b})|B_m\right][\beta B_m - C] - k = 0$, and the regulator's best response is to propose only when $B \geq B_m$.

For an arbitrary $\phi_m \in (0, 1]$ let

$$B_m(\phi_m) \equiv \frac{C}{\beta} + \frac{k}{\beta\phi_m} \tag{12}$$

be the regulator's best response function, and for an arbitrary $B_m \in (-\infty, \infty)$, let

$$\phi_m(B_m) = E_{\widetilde{b}}\left[\phi_m(\widetilde{b})|B_m\right] \tag{13}$$

be the *pivotal approval probability*, i.e., the expected probability of approval when the pivotal benefit is B_m. The best response function $B_m(\cdot)$ is identical to $B_n(\cdot)$ except that it includes k rather than k_r. Thus, $B_m(\phi) \geq B_n(\phi)$ for any fixed ϕ. The key difference between the best response functions $\phi_m(\cdot)$ and $\phi_n(\cdot)$ is that with no BCA, ϕ_n is the unconditional approval probability and does not depend on B, but with BCA, by contrast, $\phi_m(B_m)$ is the regulator's expectation of the approval probability of the marginal proposal. Note that $\phi_m(B_m) \leq E_{\widetilde{b}}\left[\phi_m(\widetilde{b})|B\right]$ for $B > B_m$ because $E_{\widetilde{b}}\left[\phi_m(\widetilde{b})|B\right]$ is non-decreasing in B.

To characterize the executive's behavior, if the executive conjectures a pivotal benefit B_m, then upon seeing a proposal resulting in a realized measured benefit b, the executive updates its belief over B as the posterior distribution of a random variable $\widetilde{B}|(b, \widetilde{B} \geq B_m)$. Now, $\widetilde{B}|b$ (i.e., *not* conditioned on $\widetilde{B} \geq B_m$) is a normal random variable with mean

$$\overline{B}_m(b) = \frac{\sigma_b^2}{\sigma_b^2 + \sigma_0^2} B_0 + \frac{\sigma_0^2}{\sigma_b^2 + \sigma_0^2} b, \tag{14}$$

and variance

$$\overline{\sigma}_m^2 = \frac{\sigma_b^2 \sigma_0^2}{\sigma_b^2 + \sigma_0^2}. \tag{15}$$

[11] This occurs if and only if there exists a set $\mathbf{b} \subseteq \mathbb{R}$ with nonzero measure such that $\phi_m(b) > 0$ for all $b \in \mathbf{b}$; i.e., the executive has a nonzero approval probability for some set of positive measure. This implies $E_{\widetilde{b}}\left[\phi_m(\widetilde{b})|B\right] > 0$ for all B because $\widetilde{b}|B \sim N(B, \sigma_b)$ has support over $\mathbb{R}$. As in the prohibited BCA subgame, our solution concept ensures this is the case.

Thus, with a pivotal benefit B_m, $\widetilde{B}|(b, \widetilde{B} \geq B_m)$ is a truncated normal random variable with mean

$$E[\widetilde{B}|b, \widetilde{B} \geq B_m] = \overline{B}_m(b) + \overline{\sigma}_m h\left(\frac{B_m - \overline{B}_m(b)}{\overline{\sigma}_m}\right).$$

For any realized b and conjectured pivotal benefit B_m, the executive's optimal approval behavior is given by

$$\phi_m(b, B_m) = \begin{cases} 1 & \text{if } \alpha E[\widetilde{B}|b, \widetilde{B} > B_m] > C \\ \in [0,1] & \text{if } \alpha E[\widetilde{B}|b, \widetilde{B} > B_m] = C \\ 0 & \text{if } \alpha E[\widetilde{B}|b, \widetilde{B} > B_m] < C \end{cases}. \tag{16}$$

That optimal approval behavior can be shown to have simple form: if the pivotal benefit B_m is sufficiently small (in particular, less than $\frac{C}{\alpha}$), the executive approves a proposal with a measured benefit that exceeds a *measured benefit threshold* b_m^T (that depends on the conjectured pivotal benefit). If the pivotal benefit is greater than or equal to $\frac{C}{\alpha}$, the executive approves the proposal irrespective of its measured benefit. We state this formally as follows.

Lemma OA2 *Suppose the executive conjectures a pivotal benefit $B_m \in (-\infty, \infty)$ and faces a measured benefit $b \in (-\infty, \infty)$. The executive's optimal approval rule $\phi_m(b, B_m)$ takes the form*

$$\phi_m(b, B_m) = \begin{cases} 1 & \text{if } b > b_m^T(B_m) \\ \in [0,1] & \text{if } b = b_m^T(B_m) \\ 0 & \text{if } b < b_m^T(B_m) \end{cases}, \tag{17}$$

where $b_m^T(B_m)$ is the executive's measured benefit threshold. This approval rule is unique over $b \neq b_m^T(B_m)$. For $B_m < \frac{C}{\alpha}$, $b_m^T(B_m) \in (-\infty, \infty)$ exists and is unique and is the solution for b to the equation

$$\overline{B}_m(b) + \overline{\sigma}_m h\left(\frac{B_m - \overline{B}_m(b)}{\overline{\sigma}_m}\right) = \frac{C}{\alpha}. \tag{18}$$

For $B_m \geq \frac{C}{\alpha}$, $b_m^T(B_m) = -\infty$, i.e., the executive will approve the proposed rule no matter what the result of the BCA.[12]

For $B_m < \frac{C}{\alpha}$, we have

$$\frac{db_m^T(B_m)}{dB_m} = -\frac{1}{\overline{B}'_m(b_m^T(B_m))} \frac{h'\left(\frac{B_m - \overline{B}m(b_m^T(B_m))}{\overline{\sigma}_m}\right)}{1 - h'\left(\frac{B_m - \overline{B}_m(b_m^T(B_m))}{\overline{\sigma}_m}\right)} < 0, \tag{19}$$

[12] We note that Lemma OA2 validates the regulator's conjecture that the approval function is nondecreasing in b and nonzero over a set of positive measure.

since $\frac{d\overline{B}_m}{db} > 0$ and $h'(z) \in (0,1)$ for all $z < 1$.[13] Thus, the larger the pivotal benefit, the smaller is the measured benefit threshold, and if the executive expects the pivotal benefit to be at least $\frac{C}{\alpha}$, the executive approves the new rule with certainty. In addition, using (18), for a given pivotal benefit, the measured benefit threshold decreases in the executive's welfare weight:

$$\frac{\partial b_m^T(B_m,\alpha)}{\partial \alpha} = \frac{-\frac{C}{\alpha^2}}{\overline{B}'_m(b_m^T(B_m,\alpha))\left[1 - h'(\frac{B_m - \overline{B}m(b_m^T(B_m,\alpha))}{\overline{\sigma}_m})\right]} < 0. \tag{20}$$

Further, it is straightforward to prove $\lim_{B_m \to \frac{C}{\alpha}} b_m^T(B_m) = -\infty$ and $\lim_{B_m \to -\infty} b_m^T(B_m) = \frac{\sigma_b^2+\sigma_0^2}{\sigma_0^2}\frac{C}{\alpha} - \frac{\sigma_b^2}{\sigma_0^2}B_0$.[14]

The equilibrium of the regulatory proposal subgame with BCA is a triple $\{B_m^*, \phi_m^*, b_m^*\}$ simultaneously satisfying Lemma OA2, (12), and (13) where, in (13),

$$\phi_m(B_m) = E_{\widetilde{b}}\left[\phi_m(\widetilde{b}, B_m)|B_m\right] = \widehat{\Psi}\left(\frac{b_m^T(B_m) - B_m}{\sigma_b}\right). \tag{21}$$

Proposition OA5 *If BCA is utilized in the regulatory process, an equilibrium in the regulatory proposal subgame, (B_m^*, ϕ_m^*, b_m^*), exists and is unique up to changes of measure zero. In that equilibrium, the regulator proposes a new rule if $B > B_m^*$ and does not propose a new rule if $B < B_m^*$. The executive accepts the proposed rule if the measured benefit b exceeds a measured benefit threshold b_m^* and rejects the proposed rule otherwise. (1) For $\alpha < \alpha_m^*(\beta) \equiv \frac{C}{C+k}\beta \in (0,\beta)$, the equilibrium is given by*

$$B_m^* = B_m(\phi_m^*) \equiv \frac{C}{\beta} + \frac{k}{\beta\phi_m^*} \in \left(\frac{C+k}{\beta}, \frac{C}{\alpha}\right). \tag{22}$$

$$\phi_m^* = \phi_m(B_m^*) = \widehat{\Psi}\left(\frac{b_m^T(B_m^*) - B_m^*}{\sigma_b}\right) \in (0,1). \tag{23}$$

$$b_m^* = b_m^T(B_m^*), \tag{24}$$

where, recall, $b_m^T(B_m)$ is the unique solution for b to (18) and ϕ_m^* is the approval probability when $B = B_m$.

[13] It is well known that $h'(z) > 0$. The proof that $h'(z) < 1$ for all $z \in (-\infty,\infty)$ is presented as part of the proof of Lemma OA2.

[14] The first limit is established in the proof of Lemma OA2. The second limit follows because when $B_m \to -\infty$, the truncated mean is simply the untruncated posterior mean, $\overline{B}_m(b)$. The measured benefit threshold then solves $\overline{B}_m(b) = \frac{C}{\alpha}$, yielding the stated limit.

(2) For $\alpha \geq \alpha_m^*(\beta)$, the equilibrium is given by

$$B_m^* = B_m(\phi_m^*) \equiv \frac{C+k}{\beta} \geq \frac{C}{\alpha}.$$

$$\phi_m^* = 1.$$

$$b_m^* = b_m^T(B_m^*) = -\infty.$$

In contrast to when BCA is not used and the equilibrium likelihood of approval is constant, the equilibrium likelihood of approval with mandatory BCA, $\widehat{\Psi}\left(\frac{b_m^*-B}{\sigma_b}\right)$, increases in the underlying benefit, and this probability approaches one as B approaches ∞. This means that the potential exists for BCA to make the executive a more efficient gatekeeper by reducing the risk of a type 1 error when it matters most—for new rules with a high B.

In the two-type model, if Assumption 2 was extended to include a non-negative direct cost of a BCA, k_a, then $\beta > \frac{C+k}{B^L}$. Because $\alpha < \frac{C}{B^L}$ in the two-type model, if would follow that $\alpha < \frac{C}{C+k}\beta = \alpha_m^*(\beta)$. Thus, the condition for part 1 of Proposition OA5 are comparable to those for Proposition 2 in the paper. Part 2 in Proposition OA5, on the other hand, does not have a counterpart in the two-type model in the paper. This is because Assumptions 1 and 2 in the main paper limit how close α and β are to each other.[15] Still, part 1 of Proposition 2 in which a sufficiently large α (i.e., $\alpha \geq \overline{\alpha}_m(b^L)$) implies that both types of the regulator propose, and the executive approves a proposal with certainty irrespective of the results of the BCA, is very much in the spirt of part 2 of Proposition OA5. Parts 2 and 3 of Proposition 2, in which the approval probability depends on the outcome of the BCA are in line with part 1 of Proposition OA5, which presents an analogous result.

When $\alpha < \alpha_m^*(\beta)$, we can use (22) and (23) to establish comparative statics results with respect to the executive's welfare weight:

$$\frac{\partial \phi_m^*}{\partial \alpha} = \frac{\sigma_b^{-1}\widehat{\Psi}'\frac{\partial b_m^T}{\partial \alpha}}{1+\sigma_b^{-1}\widehat{\Psi}'\left[\frac{\partial b_m^T}{\partial B_m}-1\right]\frac{k}{\beta}(\phi_m^*)^{-2}} > 0$$

(since $\widehat{\Psi}' < 0$, $\frac{\partial b_m^T}{\partial B_m} < 0$, and $\frac{\partial b_m^T}{\partial \alpha} < 0$), and thus from (12) and (19) $\frac{dB_m^*}{d\alpha} < 0$. Analogous to when BCA is not used, a decrease in the executive's welfare weight increases the pivotal benefit and decreases the likelihood of approval of the marginal proposal. However, the sign of $\frac{\partial b_m^*}{\partial \alpha}$ is ambiguous: from (20) a lower α increases b_m^T, which works to increase b_m^*, but a lower α also increases the equilibrium pivotal benefit, which from (19) decreases b_m^T, working to decrease b_m^*. Thus, a more regulation-averse executive might have a more lenient BCA threshold in equilibrium.

[15] However, if we relax Assumption 2 as we do in the previous section of this appendix, then part 2 of Proposition OA5 becomes comparable to the case of full alignment discussed in that section.

The value to the executive from mandatory BCA is

$$EU_m^E - EU_n^E = \int_{B_m^*}^{\infty} \left\{ \widehat{\Psi}\left(\frac{b_m^* - x}{\sigma_b}\right)[\alpha x - C] - k \right\} f_0(x)dx - \int_{B_n^*}^{\infty} \{\phi_n^* [\alpha x - C] - k_r\} f_0(x)dx. \tag{25}$$

As in the two-type model in the paper, we can decompose this value into four components, $EU_m^E - EU_n^E = \sum_{i=1}^4 \Delta_i$, where

$$\Delta_1 = \int_{\frac{C}{\alpha}}^{\infty} \left\{ \widehat{\Psi}\left(\frac{b_m^* - x}{\sigma_b}\right) I\{x \geq B_m^*\} - \phi_n^* I\{x \geq B_n^*\} \right\} [\alpha x - C] f_0(x)dx. \tag{26}$$

$$\Delta_2 = \int_{-\infty}^{\frac{C}{\alpha}} \left\{ \phi_n^* I\{x \geq B_n^*\} - \widehat{\Psi}\left(\frac{b_m^* - x}{\sigma_b}\right) I\{x \geq B_m^*\} \right\} [C - \alpha x] f_0(x)dx. \tag{27}$$

$$\Delta_3 = [F_0(B_m^*) - F_0(B_n^*)] k_r. \tag{28}$$

$$\Delta_4 = -k_a \widehat{F}_0(B_m^*). \tag{29}$$

Δ_1 is the impact of the BCA on the possibility of a type 1 error. If the approval function under BCA is a good approximation of the executive's symmetric information approval function $I\{B \geq \frac{C}{\alpha}\}$ then $\widehat{\Psi}\left(\frac{b_m^* - B}{\sigma_b}\right) \geq \phi_n^*$ over much of the range $\left[\frac{C}{\alpha}, \infty\right)$ and the use of BCA benefits the executive by reducing the possibility of a type 1 error, i.e., $\Delta_1 > 0$. However, if $\widehat{\Psi}\left(\frac{b_m^* - B}{\sigma_b}\right)$ is a poor approximation to $I\{B \geq \frac{C}{\alpha}\}$, then BCA could increase the cost to the executive from a type 1 error, i.e., $\Delta_1 < 0$.

We further note that if the executive and regulator are aligned, then Δ_1 is unambiguously negative. In this case, $\alpha > \max\{\alpha_n^*(\beta), \alpha_m^*(\beta)\}$, and thus by Propositions OA4 and OA5, we have $\phi_n^* = \widehat{\Psi}(\frac{b_m^* - B}{\sigma_b}) = 1$ for all B (since $b_m^* = -\infty$), and $B_m^* = \frac{C+k}{\beta} > \frac{C+k_r}{\beta} = B_n^* \geq \frac{C}{\alpha}$. The type 1 error term then reduces to $\Delta_1 = -\int_{B_n^*}^{B_m^*} [\alpha x - C] f_0(x)dx$. This is negative since $\alpha x - C > 0$ for all $x > \frac{C}{\alpha}$ and $B_n^* \geq \frac{C}{\alpha}$. Essentially, when the executive and regulator are aligned, the use of BCA does not change the executive's approval behavior, but it chokes off proposals that the executive would have benefited from. We note that this effect does not have a direct analogue in the two-type model: there, if the executive and regulator are aligned, the regulator will have exactly the same proposal behavior with and without BCA. However, this effect will be small if the cost of the BCA k_a is small (and therefore $B_m^* - B_n^* = \frac{k_a}{\beta}$ is small).

Δ_2 reflects the impact of the BCA on the possibility of a type 2 error. In general, its sign is ambiguous and depends on two factors: whether $\phi_n^* > \widehat{\Psi}\left(\frac{b_m^* - B}{\sigma_b}\right)$ for $B < \frac{C}{\alpha}$ and whether BCA worsens or improves selection. For example, suppose $\alpha < \alpha_m^*(\beta)$, so that $B_m^* < \frac{C}{\alpha}$. Further, suppose that BCA improves selection, i.e., $B_n^* < B_m^*$, and $\phi_n^* > \widehat{\Psi}\left(\frac{b_m^* - B}{\sigma_b}\right)$ for $B \in [B_n^*, \frac{C}{\alpha})$. It follows that for $B \in (-\infty, \frac{C}{\alpha}]$, $\phi_n^* I\{B \geq B_n^*,\} \geq \widehat{\Psi}\left(\frac{b_m^* - B}{\sigma_b}\right) I\{B \geq B_m^*\}$ (with strict inequality for $B > B_n^*$), and thus $\Delta_2 > 0$.

Δ_3 reflects the impact of BCA on *ex ante* proposal costs. This has the same sign as $B_m^* - B_n^*$. Finally,

Δ_4 reflects the incremental cost of conducting BCA. It is unambiguously negative. Δ_3 and Δ_4 indicate that BCA can serve a (costly) screening function by potentially reducing proposal costs through the regulator not proposing lower-quality rules, as well as the informational and strategic role reflected by Δ_1 and Δ_2.

Figure OA3 shows the decomposition terms when $\alpha = 0.25$ and all other parameters are at baseline levels.[16] The solid and dashed lines are the integrand of the executive's expected payoff with and without BCA, respectively—i.e., the *ex post* welfare as a function of B multiplied by the density at B in the executive's prior. Gains from BCA are areas below the dashed line and above the solid line, while losses from BCA are areas below the solid line and above the dashed line.[17] In this particular parameterization, without BCA, $B_n^* = 434.76$, $\phi_n^* = 0.014$, and with BCA, $B_m^* = 414.82$, $b_m^* = 631.53$, $\phi_m^* = 0.364$. Figure OA3 illustrates that BCA reduces the executive's welfare loss from type 1 errors but increases the welfare loss from type 2 errors. The welfare gain due to reduced type 1 errors in this case occurs primarily because over $\left[\frac{C}{\alpha}, \infty\right)$ the approval probability with BCA, $\widehat{\Psi}\left(\frac{b_m^* - B}{\sigma_b}\right)$, is larger than the approval probability without BCA, ϕ_n^*. Given the executive's priors, there is non-trivial probability mass on $\left[\frac{C}{\alpha}, \infty\right)$, so the reduction in the rate of type 1 errors matters a lot.

The welfare loss due to type 2 errors occurs because for a non-trivial range of B below $\frac{C}{\alpha}$, it continues to be more likely that the executive will approve a proposed regulation under BCA than without it. However, in this range the executive would not approve proposals if it knew B, resulting in welfare losses. This occurs within the part of the executive's prior with the greatest probability mass. On top of that, BCA worsens selection, so there is a wider range of B over which type 2 errors could occur. This worsening of selection also makes Δ_3 negative, as represented by the thin trapezoid below the horizontal axis between 414.82 and 434.76. The loss from Δ_4 cannot be easily discerned from the graph because k_a is so small, but it is a thin band along the length of the solid line.

Overall, Figure OA3 illustrates a case in which the welfare gain from reduced type 1 error under a mandated BCA more than offsets the welfare losses from increased type 2 error, increased proposal costs, and the direct cost of the BCA. Indeed, in this example, $EU_m^E - EU_n^E = 4.91 > 0$.

Summing up, the discussion above provides intuition about conditions under which the executive derives positive value from mandatory BCA:

- The approval function with BCA, $\widehat{\Psi}\left(\frac{b_m^* - B}{\sigma_b}\right)$, is larger than the approval function without BCA, ϕ_n^*, over $[\frac{C}{\alpha}, \infty)$, reducing the welfare loss from type 1 errors.
- The approval function with BCA is smaller than the approval function without BCA over $(-\infty, \frac{C}{\alpha}]$,

[16] See Table OA3.

[17] Or, in the case of Δ_3, an area left of the dashed line and right of the solid line would represent a gain from BCA, while an area right of the dashed line and left of the solid line (as in Figure OA3) would represent a loss from BCA.

reducing the welfare loss from type 2 errors.

- BCA improves selection, $B_m^* > B_n^*$, reducing the welfare loss from type 2 errors, as well as *ex ante* proposal costs.

- The direct cost of a BCA is low, and/or selection with a BCA is such that the range of rules proposed by the regulator has a low probability in the executive's prior.

Note that with a perfect BCA (i.e., $\sigma_b = 0$), $b_m^* = \frac{C}{\alpha}$ and $\widehat{\Psi}\left(\frac{b_m^* - B}{\sigma_b}\right) = I\{B \geq \frac{C}{\alpha}\}$, so a highly precise BCA should lead $\widehat{\Psi}\left(\frac{b_m^* - B}{\sigma_b}\right)$ to being a good approximation of $I\{B \geq \frac{C}{\alpha}\}$, both for $B \geq \frac{C}{\alpha}$ and $B < \frac{C}{\alpha}$. However, as in the example in Figure OA3, a noisy BCA can result in the approval function being a worse approximation to $I\{B \geq \frac{C}{\alpha}\}$ than ϕ_n^* when $B < \frac{C}{\alpha}$ but a better approximation when $B > \frac{C}{\alpha}$, decreasing the likelihood of type 1 errors but increasing the likelihood of type 2 errors. If the executive is regulation averse so that $\alpha B_0 < C$ (as in Figure OA3), and if σ_0 is relatively small (smaller than in Figure OA3), then the most of the prior probability mass will be concentrated in the range of B where type 2 errors occur. This suggests that any adverse effect of BCA on type 2 errors would be especially meaningful for a regulation-averse executive with tight priors. Supporting this intuition, if we change the example in Figure OA3 by reducing σ_0 from 100 to 40, keeping all other parameter values the same, $EU_m^E - EU_n^E = -0.00025$, giving the executive a slight preference to not use BCA.

In general, BCA could improve selection or worsen it, as the two panels of Figure OA4 illustrate. The right-hand panel illustrates equilibria for $\alpha = 0.50$ and all other parameters set to their baseline levels. In this case, BCA improves selection. The left-hand panel illustrates equilibria for $\alpha = 0.25$ and all other parameters set to their baseline values (the same case illustrated in Figure OA3). Here BCA worsens selection. The ambiguous effect of BCA on selection is consistent with our findings in the two-type model.

We conclude this section by characterizing the executive's preference for BCA in two special cases: $\alpha \geq \beta$—the executive gives at least as much weight to benefits as the regulator—and $\alpha \in \left[\frac{C}{C+k_r}\beta, \beta\right)$ so there is strong alignment between the executive and the regulator in the weighting of benefits. As mentioned in the previous discussion of the welfare decomposition, Propositions OA4 and OA5 imply $\phi_n^* = \phi_m^* = 1$, $b_m^* = -\infty$, $B_m^* = \frac{C+k}{\beta} > \frac{C+k_r}{\beta} = B_n^* > \frac{C}{\alpha}$.

Lemma OA3 *If $\alpha \geq \beta$ and $k_a > 0$, then $EU_m^E < EU_n^E$, i.e., the executive has a strict preference not to use BCA.*

The intuition is that if the executive were to mandate BCA when $\alpha \geq \beta$, two bad things would happen. First, there would be a direct cost of the BCA on any regulation it proposes, i.e., $\Delta_4 < 0$. Second, use of BCA would worsen type 1 errors: as discussed above, $\Delta_1 < 0$ in this case. While there would be no additional cost of type 2 errors ($\Delta_2 = 0$) and the proposal cost would go down, i.e., $\Delta_3 > 0$, the increased cost of a type 1 error outweighs the proposal cost savings:

$$\Delta_1 + \Delta_3 = -\int_{\frac{C+k_r}{\beta}}^{\frac{C+k}{\beta}} [\alpha x - C - k_r] f_0(x) dx < 0,$$

as $\alpha x - C - k_r > 0$ for $x > \frac{C+k_r}{\beta}$ when $\alpha \geq \beta$.

Lemma OA4 *If $\alpha > \frac{C}{C+k_r}\beta$ and $k_a > 0$ then $EU_m^E - EU_n^E$ is decreasing in α.*

In the range $\alpha > \frac{C}{C+k_r}\beta$, increases in α do not change the regulator's proposal strategy, nor do they change the executive's acceptance strategy. Therefore, they do not change the expected proposal cost or BCA cost. However, because the regulator proposes a new rule over a smaller range of potential benefits when BCA is mandated (i.e., $B_m^* > B_n^*$), as α increases, the expected type 1 error from implementing BCA increases because the executive values the foregone regulation in that range more.

Finally we characterize $EU_m^E - EU_n^E$ when $\alpha \in [\frac{C}{C+k_r}\beta, \beta]$ in a proposition that directly follows from the previous two Lemmas.

Proposition OA6 *Suppose $\alpha \in [\frac{C}{C+k_r}\beta, \beta]$. Then the following statements hold: (1) If $EU_m^E - EU_n^E < 0$ when $\alpha = \frac{C}{C+k_r}\beta$, then $EU_m^E - EU_n^E < 0$ for all $\alpha > \frac{C}{C+k_r}\beta$. (2) If $EU_m^E - EU_n^E > 0$ when $\alpha = \frac{C}{C+k_r}\beta$, then there exists $\alpha^{**}(\beta) \in (\frac{C}{C+k_r}\beta, \beta)$ such that $EU_m^E - EU_n^E > 0$ for all $\alpha \in [\frac{C}{C+k_r}\beta, \alpha^{**}(\beta))$ and $EU_m^E - EU_n^E < 0$ for all $\alpha > \alpha^{**}(\beta)$. Further, $\alpha^{**}(\beta)$ is given by*

$$\alpha^{**}(\beta) = \frac{(C+k_r)\left[F_0\left(\frac{C+k}{\beta}\right) - F_0\left(\frac{C+k_r}{\beta}\right)\right] - k_a \widehat{F}_0\left(\frac{C+k}{\beta}\right)}{B_0\left[F_0\left(\frac{C+k}{\beta}\right) - F_0\left(\frac{C+k_r}{\beta}\right)\right] + \sigma_0\left[f_0\left(\frac{C+k_r}{\beta}\right) - f_0\left(\frac{C+k}{\beta}\right)\right]}.$$

The intuition of Proposition OA6 is clearest when keeping in mind the decomposition of $EU_m^E - EU_n^E$. For sufficiently large values of α, the regulator never proposes a regulation with benefits below $\frac{C}{\alpha}$ in either subgame; as such, there are no type 2 errors in either subgame, so $\Delta_2 = 0$. In addition, for $\alpha > \frac{C}{C+k_r}\beta$, we have $\Delta_1 < 0$, $\Delta_3 > 0$, and $\Delta_4 < 0$. And as discussed following Lemma OA4, as we increase α in this range, Δ_1 decreases but Δ_3 and Δ_4 remain constant. Therefore, it is natural that if $EU_m^E - EU_n^E > 0$ for the lowest value of α in this range (i.e., $\Delta_1 + \Delta_3 + \Delta_4 > 0$), there will be a value of α where Δ_3 exactly balances $\Delta_1 + \Delta_4$ and the executive will be indifferent between mandating and prohibiting BCA. As α increases further, however, Δ_1 will decrease while Δ_3 and Δ_4 remain the same, so the executive will unambiguously prefer to not authorize BCA. This point will come at a value of α less than β.

The upshot of Proposition OA6 and Lemma OA3 is that for any welfare weight $\alpha \geq \max\{\alpha^{**}(\beta), \frac{C}{C+k_r}\beta\}$—recognizing that this threshold is strictly less than the regulator's welfare weight β—the executive prefers not to use BCA. The fact that administrations belonging to both U.S. parties have sustained the executive orders that over the last 40 years have mandated use of BCA suggests that the "normal" case is that the executive's welfare weight is below this threshold, which is consistent with presidential administrations that are more regulation averse than their regulatory agencies.

In the two-type model in the paper, Assumption 1 and 2 imply $\alpha < \frac{C}{C+k_r}\beta$. The condition $\alpha \geq \frac{C}{C+k_r}\beta$ in Proposition OA6, which implies a sufficient degree of alignment between the executive and regulator, cannot arise in our two-type model given Assumption 2 in the paper. However, as discussed in Section 3, the analog to this case does arise when we have what we call "full alignment." In that case, only a high-benefit regulator would propose a new regulation, and the executive would approve any proposal irrespective of the outcome of the BCA. Given this, the executive would be indifferent between mandating and prohibiting BCA when (as we assume in the paper) BCA is costless, and the executive would prefer not to mandate BCA if the direct cost k_a of the BCA is positive.

4.3 Regulatory proposal subgame: voluntary BCA

As in the two-type model, if BCA is voluntary, the regulator's decision to propose with versus without BCA conveys additional information. Three equilibrium configurations are possible: (a) a *maximally separating* equilibrium in which the regulator proposes with a BCA for $B \in \mathbf{B}_{BCA} \neq \emptyset$, proposes without a BCA for $B \in \mathbf{B}_{NO} \neq \emptyset$, and does not propose for $B \in \mathbb{R} \setminus (\mathbf{B}_{BCA} \cup \mathbf{B}_{NO})$; (b) a *partial pooling* equilibrium in which the regulator proposes with a BCA or does not propose at all; or (c) a partial pooling equilibrium in which the regulator proposes without a BCA or does not propose at all.[18] In cases (b) and (c), the executive's payoffs are EU_m^E and EU_n^E, respectively, just as in the top and bottom branches of the framework subgame. Allowing the regulator to voluntarily choose BCA adds a meaningful dimension to the executive's choice set only if a maximally separating equilibrium arises.

To explore a maximally separating equilibrium, consider, first, the regulator's incentives. Let $\phi_v(b)$ be the regulator's conjectured probability of approval when the executive receives a proposal with a BCA and the measured benefit is b, and let ϕ_v be the conjectured probability of approval for a proposal without a BCA. We assume $\phi_v(b)$ is nondecreasing in b and establish presently that this is consistent with optimal behavior by the executive.

Given the regulator's conjecture of the executive's strategy $(\phi_v, \phi_v(b))$, the regulator proposes without

[18] We refer to the first case as a "maximally separating equilibrium" because it is not a true separating equilibrium; the executive cannot discern between any two different values of B. However, this equilibrium offers the most separation possible to the executive in light of the lack of information available from the regulator's message established by Lemma 1 in the paper.

BCA only when

$$\phi_v(\beta B - C) - k_r \geq 0, \tag{30}$$

$$\phi_v(\beta B - C) - k_r \geq E_{\widetilde{b}}\left(\phi_v(\widetilde{b})|B\right)(\beta B - C) - k. \tag{31}$$

It proposes with a BCA only when

$$E_{\widetilde{b}}\left(\phi_v(\widetilde{b})|B\right)(\beta B - C) - k \geq 0, \tag{32}$$

$$E_{\widetilde{b}}\left(\phi_v(\widetilde{b})|B\right)(\beta B - C) - k \geq \phi_v(\beta B - C) - k_r, \tag{33}$$

and it does not propose for B that do not satisfy (30) or (32). These conditions, coupled with optimizing behavior by the executive, imply that if a maximally separating equilibrium exists, it must have the following properties.

Lemma OA5 *If a maximally separating equilibrium exists, then there must be pivotal benefits B_v^- and B_v^+, with $B_v^- < B_v^+$, such that*

$$B_v^- = \frac{C}{\beta} + \frac{k_r}{\beta\phi_v}, \tag{34}$$

$$B_v^+ = \frac{C}{\beta} + \frac{k_a}{\beta(1-\phi_v)}, \tag{35}$$

where the regulator does not propose if $B < B_v^-$, proposes a new regulation without a BCA if $B \in (B_v^-, B_v^+)$, and proposes a new regulation supported by a BCA if $B > B_v^+$. (At the boundaries B_v^- and B_v^+, the proposal strategy must place positive probability only on not proposing/proposing without BCA and on proposing with/without BCA respectively). Any proposal accompanied by a BCA is approved with certainty, i.e., the approval rule is $\phi_v(b) = 1$ for all $b \in (-\infty, \infty)$. The approval probability for a proposal without a BCA is such that $\phi_v \in \left(\frac{k_r}{k}, 1\right)$. Further, a necessary condition for a maximally separating equilibrium to arise is $k_a > 0$.

Lemma OA5 tells us that if a maximally separating equilibrium in the voluntary BCA subgame exists, it must involve a low range of B over which the regulator does not propose a new regulation, an intermediate range over which the regulator proposes without a BCA, and a high range over which the regulator proposes with a BCA. Further, a striking feature of the equilibrium is that BCA plays no role as a noisy signal of the underlying benefit. Instead, submission of a BCA with a proposed rule is a signal that the underlying benefit is sufficiently high, and given that, the executive ignores the measured benefit provided by the BCA and approves the rule with certainty. Intuitively, the executive screens higher B proposals ($B \geq B_v^+$) from lower B proposals ($B \in [B_v^-, B_v^+)$) by making the (proposal cost, approval probability) allocation $(k_r + k_a, 1)$ more attractive than (k_r, ϕ_v) as illustrated in Figure OA5 which shows the regulator's welfare U^R as a function of the benefit B.

If a maximally separating equilibrium exists, it is characterized by pivotal benefits B_v^- and B_v^+ and an approval probability $\phi_v \in (\frac{k_r}{k}, 1)$ such that (34) and (35) hold and

$$\alpha E\left[\widetilde{B}|\widetilde{B} \in \left[\frac{C}{\beta} + \frac{k_r}{\beta\phi_v}, \frac{C}{\beta} + \frac{k_a}{\beta(1-\phi_v)}\right]\right] = C. \tag{36}$$

If a solution to (36) exists and is such that $\phi_v \in (\frac{k_r}{k}, 1)$, then a separating equilibrium exists.

We now establish necessary and sufficient conditions for such an equilibrium to exist.

Proposition OA7 *Suppose $k_a > 0$ and let*

$$\overline{B}^{\min} = \min_{\phi \in \left[\frac{k_r}{k}, 1\right]} \overline{B}(\phi),$$

$$\overline{B}^{\max} = \max_{\phi \in \left[\frac{k_r}{k}, 1\right]} \overline{B}(\phi),$$

where

$$\overline{B}(\phi) \equiv E\left[\widetilde{B}|\widetilde{B} \in \left[\frac{C}{\beta} + \frac{k_r}{\beta\phi}, \frac{C}{\beta} + \frac{k_a}{\beta(1-\phi)}\right]\right]. \tag{37}$$

For any $\alpha \in \left(\frac{C}{\overline{B}^{\max}}, \frac{C}{\overline{B}^{\min}}\right)$ there exists a separating equilibrium in the voluntary BCA subgame characterized by a triple, $\{B_v^{-*}, B_v^{+*}, \phi_v^*\}$, satisfying (34), (35), (36), and $\phi_v^* \in \left(\frac{k_r}{k}, 1\right)$. If $\alpha < \frac{C}{\overline{B}^{\max}}$ or $\alpha > \frac{C}{\overline{B}^{\min}}$, then a maximally separating equilibrium does not exist. A maximally separating equilibrium exists for $\alpha = \frac{C}{\overline{B}^{\min}}$ unless $\frac{C}{\overline{B}^{\min}} = \alpha_n^*(\beta)$ or $\frac{C}{\overline{B}^{\min}} = \alpha_m^*(\beta)$, and a maximally separating equilibrium exists for $\alpha = \frac{C}{\overline{B}^{\max}}$ unless $\frac{C}{\overline{B}^{\max}} = \alpha_n^*(\beta)$ or $\frac{C}{\overline{B}^{\max}} = \alpha_m^*(\beta)$.

Letting $\Phi(B|\phi_v^*, B_v^{-*}, B_v^{+*}) \equiv \phi_v^* I\{B \geq B_v^{*-}\} + (1-\phi_v^*) I\{B \geq B_v^{*+}\}$, the value of the executive from allowing voluntary BCA as opposed to not using BCA is

$$EU_v^E - EU_n^E = \int_{B_v^{*-}}^{\infty} \left\{\Phi(x|\phi_v^*, B_v^{*-}, B_v^{+*})\left[\alpha x - C\right] - k\right\} f_0(x)dx - \int_{B_n^*}^{\infty} \left\{\phi_n^*\left[\alpha x - C\right] - k_r\right\} f_0(x)dx.$$

As with mandatory BCA, this welfare difference can be decomposed into four components analogous to those in (26)-(29), except that the decomposition uses the approval function $\Phi(B|\phi_v^*, B_v^{-*}, B_v^{+*})$ instead of $\widehat{\Psi}\left(\frac{b_m^* - B}{\sigma_b}\right)$.

The possible existence of a maximally separating equilibrium is in contrast with our finding in the two-type model that such an equilibrium does not exist. We also showed in Proposition 8 of the paper that in the general model with $k_a = 0$ is outcome equivalent to an equilibrium in which BCA is prohibited. We see this phenomenon in our continuous-type model in this appendix. When voluntary BCA in that case gives rise to a maximally separating equilibrium and the direct cost k_a of a BCA is small relative to overall proposal costs, the approval probability ϕ_v^* for a proposal without a BCA will be close to one, and $\Phi(B|\phi_v^*, B_v^{-*}, B_v^{+*})$ will either equal one or be close to it for all $B > B_v^{*-}$. Under these circumstances, accompanying a proposal with a BCA is a relatively inexpensive signal for the regulator. Therefore, the executive's approval probability for a proposal without a BCA has to be sufficiently high in order to induce the regulator not to submit with a BCA when the underlying benefit of the regulation is middling. A maximally separating equilibrium in the voluntary BCA subgame is therefore substantially similar to an equilibrium with $\phi_n^* = 1$ and $B_n^* = \frac{C+k_r}{\beta}$ in the prohibited BCA subgame. As such, the executive will tend to implement voluntary BCA only when (a) $\phi_n^* \approx 1$ but (b) the executive is still subject to a somewhat substantial probability of type 2 error, meaning it gains more from the small improvement in selection and increased rejection probability than from the cost of implementing a BCA for a portion of proposals. We present further evidence of this observation momentarily and show that it only corresponds to a narrow range of parameters.

Finally, notice that there may be multiple maximally separating equilibria. As ϕ increases, the lower endpoint of the truncation in (37) decreases and the upper endpoint of the truncation increases, meaning the change in the expression as a whole is unclear. Therefore, there may be multiple values of $\phi \in (\frac{k_r}{k}, 1)$ satisfying $\alpha\overline{B}(\phi) - C = 0$.

4.4 Computational analysis

To make further headway in analyzing the executive's equilibrium choice of a regulatory framework we turn to computational analysis. Computational analysis is useful because key objects in the analysis, such as the difference in the executive's welfare with mandatory versus prohibited BCA $EU_m^E - EU_n^E$, do not have unambiguous signs or comparative statics with respect to underlying parameters. The computational analysis is based on the baseline parameterization and the associated parameter ranges in Table OA3.

The intent of the computational analysis is not to simulate any particular case study but rather to identify regularities among empirically plausible values. Consistent with that objective, we note that the baseline prior benefit-cost ratio $\frac{B_0}{C}$ of 3 to 1 falls within the range of the Environmental Protection Agency's (2011) assessment of the benefit-cost ratio from the Clean Air Act. The ranges of B_0 and C trace out benefit-cost ratios ranging from $\frac{1}{3}$ to 20, the latter value being at the high end of the EPA's assessment.

Parameter	Baseline values	Range
B_0, expected net benefit of proposed rule	\$450	\$150, \$300, \$450, \$600, \$1,000
C, expected compliance cost	\$150	\$50, \$100, \$150, \$300, \$450
σ_0, imprecision of executive's prior beliefs	\$200	\$20, \$40, \$80, \$120, \$160, ..., \$400, \$1,000
σ_b, noisiness of BCA	\$100	\$5, \$20, \$40, \$60, ..., \$200, \$400, \$1,000
β, regulator's welfare weight	1.0	0.40, 0.60, 0.80, 1.00, 1.20
α, executive's welfare weight	0.25, 0.50	increments of 0.0125 from 0.1 to 1.20
k_a, cost of conducting BCA	\$0.002	\$0.002, \$0.01, \$0.5
k_r, cost of developing proposed rule	\$4	\$0.01, \$4, \$50

Table OA3: Baseline parameterization for computational analysis. All monetary units are in billions.

The baseline value $\sigma_0 = \$200$ billion implies moderate uncertainty of the possible effects of the regulation for the executive. Using the baseline values of B_0 and C, it implies that the executive is 95 percent confident that the net social benefit $\widetilde{B}$ falls between \$58 billion and \$842 billion. The baseline value $\sigma_b = 100$ implies a BCA twice as precise as the executive's prior. It is also broadly consistent with the Monte Carlo analysis undertaken by National Highway Traffic and Safety Administration (NHSTA) in its benefit-cost analysis of the Obama-era fuel economy standards (National Highway Traffic Safety Administration 2011).

The baseline value $\beta = 1$ implies that the regulator strives to maximize social welfare, a natural benchmark. Given the baseline values of C, B_0, σ_0, and k, a welfare weight $\beta = 1$ corresponds to a regulation-sympathy index of about 0.931, making the regulator solidly regulation sympathetic. We vary β from 0.4 to 1.20, moving the regulator from somewhat regulation sympathetic (regulation-sympathy index of 0.627) to strongly regulation sympathetic (regulation-sympathy index of 0.946) using baseline values of B_0 and C. The regulator may also be regulation averse using other values of B_0 and C in our parameter grid.

By varying α between 0.1 and 1.20, we allow for cases where the executive is more regulation sympathetic than, more regulation averse than, and similarly disposed to regulation as the regulator. We note that our lower bound of the executive's welfare weight, $\alpha = 0.10$, implies an extremely regulation-averse administration: fixing other parameters at their baseline levels, $\widehat{F}_0\left(\frac{C}{\alpha}\right) = 7.60 \times 10^{-8}$, so the executive's prior is to reject effectively all regulation. By contrast, at the baseline values $\alpha = 0.25$ and $\alpha = 0.50$, the regulation-sympathy indices are 0.227 and 0.773 respectively. A welfare weight $\alpha = 0.25$ reflects a somewhat regulation-averse, center-right administration, while $\alpha = 0.50$ is consistent with a regulation-sympathetic, center-left administration that recognizes the political reality of the need to balance broader societal gains from regulation with their impact on business.

We set the baseline value of k_a to \$0.002 billion. In practice, BCAs done by U.S. federal agencies can take one or two years to complete. (For example, the BCA for the Obama-era fuel economy standards took roughly two years to complete.) Our baseline value reflects a total hourly labor cost of \$500, incurred over

4,000 work-hours. However, we also let k_a also take on a values of \$0.01 billion and \$0.5 billion, which would reflect hourly labor costs of \$2,500 and \$125,000 (or, equivalently, hourly labor costs of \$500 incurred over 20,000 and 1,000,000 work-hours respectively).

To determine an empirically plausible baseline value of k_r, recall that k_r reflects not just direct outlays by regulatory agencies (which, for a complex new rule like fuel economy standards might be upwards of \$1 billion), but also opportunity costs to the regulator of focusing on the development of one particular new regulation at the expense of other activities. It seems plausible that k_r is significantly greater than k_a but still an order of magnitude less than C. Our baseline value of k_r = \$4 billion (about 2.7 percent of the baseline value of C) reflects this intuition. However, we also allow k_r to take on a significantly lower value (\$0.1 billion) and a significantly higher one (\$50 billion) to reflect our uncertainty about this parameter.

All together, the Cartesian product of the ranges in Table OA3 imply 15,619,500 distinct parameterizations. We use Julia as the programming language for our computations, and the computations were executed on the Quest supercomputer cluster at Northwestern University.

We divide the remainder of this section into two parts. We first characterize the executive's optimal choice of a regulatory framework. We then show how key parameters in the model affect the executive's value of mandatory BCA as opposed to no BCA, $EU_m^E - EU_n^E$.[19]

4.4.1 The executive's optimal regulatory framework

We begin by observing that voluntary BCA is rarely the optimal choice for the executive.

Result 1 *The executive rarely prefers voluntary BCA to a BCA prohibition or mandatory BCA.* $EU_v^E > \max\{EU_n^E, EU_m^E\}$ *in just 0.026% of parameterizations. Further, in every parameterization where voluntary BCA is optimal,* $\phi_n^* = 1$*, i.e., without BCA the executive would approve any rule proposed by the regulator.*[20]

Although the non-existence of a maximally separating equilibrium in the two-type model is special to that case, Result 1 highlights that we do not lose much generality with respect to voluntary BCA by featuring the two-type model in the main paper.

In light of Result 1, we limit the remainder of our discussion to the comparison between prohibited BCA and mandatory BCA. Table OA4 summarizes this comparison.[21] In 71.09% of parameterizations,

[19] Some of the figures and statistics presented below integrate over the entire parameter space, e.g. to calculate averages or quantiles of certain equilibrium values. When doing so, we typically weight different parameterizations in a uniform manner, i.e., we do not weight parameterizations with parameters closer to our baseline higher. We do so not because we believe that the underlying distribution of these parameters is uniform among the points in our parameter grid, but to demonstrate that our conclusions hold over a wide range of parameter values.

[20] In our computational analysis, the case of multiple maximally separating equilibra is rare. Of the 1,201,500 unique parameterizations relevant for voluntary BCA (fewer than the 15.6 million parameterizations since the equilibrium outcomes of the voluntary BCA subgame do not depend on σ_b), in 776,710 parameterizations a maximally separating equilibrium did not arise, 424,731 had a single maximally separating equilibrium, and just 59 had two maximally separating equilibria.

[21] In the last row of Table OA4, the BCA makes the executive a tougher (softer) gatekeeper if the probability of approval without BCA is greater than (less than) the probability of approval with BCA *were the executive to believe the pivotal benefit*

	All parameterizations		$\alpha < \beta$ only	
	BCA prohibited	BCA mandated	BCA prohibited	BCA mandated
Percentage of parameterizations	71.09%	28.89%	54.05%	45.91%
mean $EU_m^E - EU_n^E$	−\$0.13	\$3.88	−\$0.13	\$3.88
mean Δ_1	−\$0.015	\$0.805	\$0.005	\$0.805
mean Δ_2	\$0.003	\$2.339	\$0.005	\$2.339
mean Δ_3	−\$0.004	\$0.795	−\$0.013	\$0.795
mean Δ_4	−\$0.113	−\$0.055	−\$0.130	−\$0.055
BCA improves selection?	90.83%	87.75%	80.84%	87.75%
BCA makes executive a tougher gatekeeper?	15.58%	86.70%	32.56%	86.70%

Table OA4: Summary of computations across all parameterizations. All monetary units are billions

$EU_n^E = \max\{EU_n^E, EU_m^E, EU_v^E\}$, so the executive would prohibit BCA. In 28.89% of the parameterizations $EU_m^E = \max\{EU_n^E, EU_m^E, EU_v^E\}$, resulting in the executive mandating BCA. If we restrict attention to parameterizations in which the executive is more regulation averse than the regulator, these percentages become 54.05% and 45.91%, respectively.[22] There are three notable asymmetries between the parameterizations giving rise to these two outcomes.

Result 2 *The gain from mandatory BCA to the executive when mandating BCA is optimal tends to be much larger than the loss from mandatory BCA when prohibiting BCA is optimal: when the executive prefers to mandate BCA, $EU_m^E - EU_n^E$ is \$3.88 billion on average. When the executive prefers to prohibit BCA, $EU_n^E - EU_m^E$ is \$0.13 billion on average.*

Result 3 *When mandating BCA is optimal, on average, about 60% of the executive's gain from mandatory BCA comes from reduced welfare loss from type 2 errors ($\Delta_2 > 0$) and about 21% of the gain comes from reduced welfare loss from type 1 errors ($\Delta_1 > 0$). When prohibiting BCA is optimal, about 87% of the loss on average from mandating BCA comes from the direct cost of the BCA itself ($\Delta_4 < 0$).*

Result 4 *BCA improves selection in about 90% of parameterizations, irrespective of whether the executive prefers mandating or prohibiting BCA. However, when the executive prefers to mandate BCA, BCA tends to make the executive a tougher gatekeeper than when the executive prefers to prohibit BCA: in about 86% of parameterizations in which the executive prefers mandatory BCA, BCA made the executive a tougher gatekeeper. In about 16% of parameterizations in which the executive prefers that BCA not be used, BCA made the executive a tougher gatekeeper.*

The asymmetry highlighted in Result 2 is akin to the asymmetry identified by Proposition 7 in the main paper for the two-type case.

remained fixed at B_n^*, i.e., $\phi_n^* > (<)\ \phi_m(B_n^*) = \widehat{\Psi}\left(\frac{b_m^T(B_n^*) - B_n^*}{\sigma_b}\right)$.

[22] Note that $\alpha \geq \beta$ in 36.6% of parameterizations. As Lemma OA3 shows, all of these cases feature $EU_n^E > EU_m^E$.

The association highlighted in Result 4 between the value of BCA to the executive and its propensity to make the executive a tougher gatekeeper is consistent with the arguments made by advocates of BCA as antidote to over-regulation in the years just before Executive Order 12291 was issued. The finding in Result 3 that the majority of the gains from BCA when it is beneficial come from reducing welfare losses from type 2 errors is consistent with this perspective. However, it is also noteworthy that not all the welfare gain from BCA comes from reducing losses from type 2 errors or excessive proposal costs. As Result 3 indicates, about one-fifth of the gain comes from reduction in welfare losses from type 1 errors. The notion that BCA can make it more likely that the executive will accept "good" proposals is consistent with our findings in the two-type model and our discussion of that finding in relationship to Revesz and Livermore (2008).

4.4.2 Determinants of the value of mandatory BCA

We now turn to the impact of specific parameters on the executive's value from mandatory BCA. Overall, BCA is valuable to the executive only to the extent that the information provided by it is useful in changing the equilibrium outcome. We focus on the impact of the following parameters: σ_b, the precision of the BCA; σ_0, the precision of the executive's priors; and the interaction between $\widehat{F}_0\left(\frac{C}{\alpha}\right)$ and $\widehat{F}_0\left(\frac{C+k}{\beta}\right)$, the regulation-sympathy indices of the executive and regulator.

Noisiness of the BCA Figure OA6 shows a boxplot of $EU_m^E - EU_n^E$ as a function of σ_b.

Result 5 $EU_m^E - EU_n^E$ *tends to be larger the more precise the BCA.*

Result 5 is intuitive. The noisier the BCA, the less information it conveys. We can see this through a statistic, $\sigma_0^2 - \overline{\sigma}_m^2$, that captures the *added precision* of the BCA—the reduction in variance of the executive's distribution of $\widetilde{B}$ that arises from BCA. Notice that $\frac{\partial}{\partial \sigma_b^2}(\sigma_0^2 - \overline{\sigma}_m^2) = -\frac{\sigma_0^4}{\sigma_0^4 + 2\sigma_0^2\sigma_b^2 + \sigma_b^4} < 0$. The increase in imprecision inhibits the executive's ability to use BCA to help prevent type 1 and type 2 errors.

Imprecision of the executive's priors Figure OA7 shows a boxplot of $EU_m^E - EU_n^E$ as a function of the standard deviation of the prior distribution, σ_0.

Result 6 $EU_m^E - EU_n^E$ *tends to be larger the less precise the executive's priors about the benefit of a regulation.*

There are two intuitive explanations for this. The more immediate intuition is similar to the one for σ_b: looser priors for the executive means that the executive gains relatively more information from BCA. In fact, an increase in σ_0 will also make the executive's posterior distribution after receiving the BCA $\overline{\sigma}_m$ larger, but by less than the amount of increase of σ_0 itself: $\frac{\partial}{\partial \sigma_0^2}(\sigma_0^2 - \overline{\sigma}_m^2) = 1 - \frac{\sigma_b^4}{\sigma_0^4 + 2\sigma_0^2\sigma_b^2 + \sigma_b^4} > 0$. As such, an increase in σ_0 will increase the value added by the signal from the BCA.

The more subtle intuition is that an increase in σ_0 flattens the executive's prior distribution of $\widetilde{B}$, increasing the probability of more extreme values of B for which reductions in type 1 and type 2 errors become more important. Recall from Table OA4 that when the executive tends to prefer BCA, most of its value is derived from reductions in type 1 and type 2 errors. As σ_0 increases, more weight is given to very high and very low values of $\widetilde{B}$ in the executive's prior. When calculating the executive's *ex ante* utility, it is therefore more important to classify rules on the fringes of the executive's prior distribution correctly. Thus, the benefit of the BCA in reducing type 1 and/or type 2 errors is amplified.

Interaction between the executive's and regulator's regulation aversion Figure OA8 shows a heatmap with the executive's regulation-sympathy index on the horizontal axis and the regulator's regulation-sympathy index on the vertical axis. In developing this map, all parameters vary over their entire ranges. In the companion Figure OA9, we present the same heatmap, but we fix σ_b at three values: its baseline value of 100 and the highest and lowest values of σ_b in our parameter range, 5 and 1,000.

Result 7 *A necessary condition for $EU_m^E - EU_n^E > 0$ is that the executive is more regulation averse than the regulator, i.e., $\alpha < \frac{C}{C+k}\beta$.*

This result is broadly consistent with Proposition OA6, and the intuition (discussed above) that underlies it. It is also consistent with our findings in the two-type model, although in that case the necessary conditions for $EU_m^E - EU_n^E > 0$ do not boil down to the single inequality $\alpha < \frac{C}{C+k}\beta$.

Result 8 *$EU_m^E - EU_n^E$ tends to be highest when $\widehat{F}_0\left(\frac{C}{\alpha}\right)$ ranges between 0.15 and 0.55—i.e., when the executive is solidly regulation averse to roughly regulation neutral—and when the corresponding $\widehat{F}_0\left(\frac{C+k}{\beta}\right)$ exceeds $\widehat{F}_0\left(\frac{C}{\alpha}\right)$ by approximately 0.25 to 0.40—i.e., when the regulator is somewhat more regulation sympathetic than the executive.*

To build intuition for Result 8, suppose that the executive is roughly regulation-neutral: $\widehat{F}_0\left(\frac{C}{\alpha}\right) \approx 0.5$ (or, equivalently, $B_0 \approx \frac{C}{\alpha}$). This implies that a large portion of the executive's prior distribution of $\widetilde{B}$ is above the cutoff above which the executive would like to approve proposals, $\frac{C}{\alpha}$, and a large portion of the distribution is below $\frac{C}{\alpha}$, where the executive would like to reject. However, if there is an agency problem between the executive and the regulator (which arises when $\alpha < \frac{C}{C+k}\beta$), the regulator would prefer to propose a wider range of rules than the executive would like to accept. Without BCA, this puts the executive in a bind. The executive could either reduce its approval probability, which would reduce proposal costs through improved selection and also reduce the type 2 error probability but at the cost of a higher likelihood of a type 1 error; or it could increase its approval probability, reducing the likelihood of a type 1 error at the expense of increasing the probability of a type 2 error and increasing proposal costs. If the executive is close to regulation neutral, both type 1 and type 2 errors are distinct possibilities, so the executive places a high value on the signal provided by BCA because it enables the executive to more effectively distinguish between instances of $\widetilde{B}$ above or below $\frac{C}{\alpha}$.

We see this value in the top left and top right panels of Figure OA10. These show the deadweight loss and welfare gain decomposition components when $\alpha = 0.3375$ and all other parameters are fixed at the baseline parameterization, implying $\widehat{F}_0(\frac{C}{\alpha}) = 0.511$ and $\widehat{F}_0(\frac{C+k}{\beta}) = 0.930$. Without BCA, there is significant welfare loss to the executive from both type 1 and type 2 errors. With BCA, the executive can achieve a big reduction in welfare loss from type 2 error without substantially increasing welfare loss from type 1 errors. (Indeed, in this case $\Delta_2 = 15.68$, while $\Delta_1 = -1.29$.)

Now suppose instead the executive is very regulation sympathetic: $\widehat{F}_0\left(\frac{C}{\alpha}\right) \gg 0.5$. (The bottom two panels in Figure OA10 pertain to this case, in which $\alpha = 0.3375$, $B_0 = \$1{,}000$, and all other parameters are held at baseline levels, which implies $\widehat{F}_0\left(\frac{C}{\alpha}\right) = 0.997$ and $\widehat{F}_0\left(\frac{C+k}{\beta}\right) = 0.999$.) Almost all of the mass of the distribution of $\widetilde{B}$ is above $\frac{C}{\alpha}$. In this case, the *ex ante* probability of a type 2 error is very small, so even without BCA, the executive can confidently approve almost any proposal it receives from the regulator. (And indeed, $\phi_n^* = 1$ in this case). BCA might help weed out those few cases in which $\widetilde{B} < \frac{C}{\alpha}$, but these are low probability occurrences, so mandatory BCA hardly adds any value for the executive: $EU_m^E - EU_n^E = 0.013$.

Finally, suppose $\widehat{F}_0\left(\frac{C}{\alpha}\right) \ll 0.5$, so we have an extremely regulation-averse executive. (The two middle panels in Figure OA10 pertain to this case, in which $\alpha = 0.3375$, $B_0 = \$150$, and all other parameters are at baseline levels, which implies $\widehat{F}_0\left(\frac{C}{\alpha}\right) = 0.070$ and $\widehat{F}_0\left(\frac{C+k}{\beta}\right) = 0.492$.) With much of the mass of the distribution of $\widetilde{B}$ below $\frac{C}{\alpha}$, the *ex ante* probability of a type 1 error is fairly small. Without BCA, the executive can confidently reject almost any proposal it receives from the regulator. (And indeed, in this case, $\phi_n^* = 0.02$.) BCA does allow the executive to approve more proposals, reducing the welfare loss from type 1 errors, but this effect is small because it is happening away from where much of the mass of the executive's prior distribution is located. Further, the use of BCA worsens selection, increasing the welfare loss from type 2 errors and higher expected proposal costs. (In this case, $\Delta_1 = 1.58$, $\Delta_2 = -0.24$, and $\Delta_3 = -0.17$; note that the scale of the vertical axis in the middle panels is an order of magnitude smaller than those in the other panels.)

Result 9 *Holding α fixed, $EU_m^E - EU_n^E$ does not necessarily increase in the the degree to which the regulator's welfare weight β diverges from the executive's.*

Result 9 is consistent with part 4 of Lemma 4 for the two-type model in the paper, which implies that BCA is more valuable to the executive when the misalignment between the executive and regulator is moderate than it is when the misalignment is either severe or modest.

The intuition underlying Result 9 is bound up in the nature of the selection problem created by the divergence between the executive's and regulator's welfare weights. If the regulator is significantly more regulation sympathetic than the executive, i.e., $\widehat{F}_0\left(\frac{C+k}{\beta}\right) \gg \widehat{F}_0\left(\frac{C}{\alpha}\right)$, the regulator's preferred proposal range differs so much from the executive's that without BCA, the executive is nearly certain to reject any proposed rule. Even if BCA improves selection, the regulator's proposal range is likely to be far outside where the executive would prefer to approve. Further, it is unlikely that BCA will makes the executive a tougher gatekeeper, because without BCA, the executive is about as tough a gatekeeper as it can be: as just noted, it would reject any proposed regulation with near certainty.

Result 10 *Variation in σ_b affects the magnitude of the executive's valuation of BCA, $EU_m^E - EU_n^E$, but it does not substantially change how the interaction of $\widehat{F}_0\left(\frac{C}{\alpha}\right)$ and $\widehat{F}_0\left(\frac{C+k}{\beta}\right)$ affects $EU_m^E - EU_n^E$.*

See Figure OA9. The upshot of this result is that the value the executive derives from BCA in our model is driven by two forces that operate largely independently: the noisiness of the BCA and the regulation aversion of the executive and regulator.

4.5 Discretionary BCA with bias: continuous-type model

Analogous to the model in the paper, we model bias by assuming that the executive commits to a methodology such that

$$\widetilde{b}_a = B + A + \sigma_b \widetilde{\varepsilon}_b,$$

where A is the degree of bias in the measurement of net benefits. When $A > 0$ the measurement of net benefits is biased upward, and when $A < 0$ measured benefits exclude a component of true benefits. Throughout, we assume that the standard deviation σ_b of the measured benefit is independent of A.

If the regulator has discretion in how it uses BCA, the equilibrium in the regulatory approval subgame is given by a triple $\{B_a^*, \phi_a^*, b_a^*\}$ simultaneously satisfying

$$\begin{aligned} B_a^* &= B_a(\phi_a^*) \equiv \frac{C}{\beta} + \frac{k}{\beta \phi_a^*}. \\ \phi_a^* &= \phi_a(B_a^*) = \widehat{\Psi}\left(\frac{b_a^* - B_a^* - A}{\sigma_b}\right). \\ b_a^* &= b_a^T(B_a, A), \end{aligned}$$

where

$$b_a^T(B_a, A) = \begin{cases} -\infty & \text{if } B_a \geq \frac{C}{\alpha} \\ \text{solution to: } \overline{B}_a(b, A) + \overline{\sigma}_a h(\frac{B_a - \overline{B}_a(b,A)}{\overline{\sigma}_a}) = \frac{C}{\alpha} & \text{if } B_a < \frac{C}{\alpha}. \end{cases}$$

and

$$\overline{B}_a(b, A) = \frac{\sigma_b^2}{\sigma_b^2 + \sigma_0^2} B_0 + \frac{\sigma_0^2}{\sigma_b^2 + \sigma_0^2} (b_a - A)$$

is the mean of the posterior distribution of $\widetilde{B}$ conditional on a realization $b_a - A$ of the de-biased BCA signal $\widetilde{b}_a - A$.

Consistent with the two-type model, if the executive has discretion in how it uses the results of the BCA, the extent of bias is irrelevant: the regulator de-biases the BCA and acts exactly as it would in Section 4.2.

Proposition OA8 *If BCA is mandatory with bias A, then the equilibrium in the regulatory approval subgame is identical to that when the BCA is mandatory and unbiased, except that the measured benefit threshold is adjusted by the magnitude of the bias, i.e.,*

$$B_a^* = B_m^*$$

$$\phi_a^* = \phi_m^*$$

$$b_a^* = b_m^* + A.$$

4.6 Strict BCA with bias: continuous-type model

As in the main paper, we assume the executive commits to accepting a proposed rule if and only if $\widetilde{b}_a \geq C$, or equivalently $\widetilde{b} \geq C - A$, where $\widetilde{b}$ is an unbiased BCA. The executive's approval behavior is described by

$$a_{sa}(b) = \begin{cases} 1 & b \geq C - A \\ 0 & \text{otherwise} \end{cases},$$

Knowing B, the regulator anticipates an approval probability $\phi_{sa}(B) = E_{\widetilde{b}}\left[a_s(\widetilde{b})|B\right] = \widehat{\Psi}\left(\frac{C-A-B}{\sigma_b}\right)$, which strictly increases in B. The regulator's expected benefit from proposing a new rule is thus $\phi_{sa}(B)(\beta B - C) - k$. On the relevant range $B > \frac{C}{\beta}$, this is strictly increasing in B, and $\lim_{B\to\infty} \phi_{sa}(B)(\beta B - C) - k = \infty$ while $\lim_{B\to\frac{C}{\beta}} \phi_{sa}(B)(\beta B - C) - k = -k$. Thus, there exists a unique pivotal benefit $B_{sa}(A)$ such that

$\phi_{sa}(B_{sa}(A))(\beta B_{sa}(A) - C) - k = 0$, or equivalently

$$B_{sa}(A) = \frac{C}{\beta} + \frac{k}{\beta \widehat{\Psi}\left(\frac{C-A-B_{sa}(A)}{\sigma_b}\right)}. \tag{38}$$

Note that $\lim_{A\to\infty} B_{sa}(A) = \frac{C+k}{\beta}$, and $\lim_{A\to-\infty} B_{sa}(A) = \infty$. The latter limit implies that with an infinite degree of bias toward understating net benefits, the regulator will not propose a new rule.

The executive's optimal degree of bias solves the optimization problem

$$EU^E_{sa} = \max_{A\in\overline{\mathbb{R}}} EU^E(A,\alpha) = \max_{A\in\overline{\mathbb{R}}} \int_{B_{sa}(A)}^{\infty} \left\{ \widehat{\Psi}\left(\frac{C-A-x}{\sigma_b}\right)[\alpha x - C] - k \right\} f_0(x)dx. \tag{39}$$

The optimization is over the set of extended real numbers, which means that $A = \infty$ or $A = -\infty$ are feasible solutions.[23] Because $\overline{\mathbb{R}}$ is a compact set, a solution to this optimization problem exists, though it is not necessarily unique.

From (38), we have $B_{sa}(C - b^*_m) = B^*_m$. Thus, through the appropriate choice of A, the executive can attain the outcome under mandated BCA with discretion, and thus $EU^E_{sa} \geq EU^E_m$. Another possible outcome of the regulator's design problem is $A = 0$. This is a strict BCA standard in its purest form.

We can express $\frac{\partial EU^E(A,\alpha)}{\partial A}$ as the difference between a marginal benefit and marginal cost, i.e., $\frac{\partial EU^E(A,\alpha)}{\partial A} = MB(A,\alpha) - MC(A,\alpha)$ where

$$MB(A,\alpha) = \int_{B_{sa}(A)}^{\infty} \psi\left(\frac{C-A-x}{\sigma_b}\right)(\frac{1}{\sigma_b})[\alpha x - C]f_0(x)dx. \tag{40}$$

$$MC(A,\alpha) = (\beta-\alpha)\frac{kB_{sa}(A)}{\beta B_{sa}(A) - C} f_0(B^*_{sa}(A))\left(-\frac{dB_{sa}(A)}{dA}\right), \tag{41}$$

and $\frac{dB_{sa}(A)}{dA}$ is obtained from (38):

$$\frac{dB_{sa}(A)}{dA} = -\frac{[\beta B_{sa}(A) - C]\, h\left(\frac{C-A-B_{sa}(A)}{\sigma_b}\right)}{\sigma_b\beta + [\beta B_{sa}(A) - C]\, h\left(\frac{C-A-B_{sa}(A)}{\sigma_b}\right)} \in (-1, 0).$$

The marginal benefit of increasing A comes from a reduced welfare loss to the executive from type 1 errors.

[23] There are several justifications for allowing the executive to introduce infinite bias into its BCA methodology. First, we can think of $A = \pm\infty$ as the limit of large finite degrees of bias. Another possible justification is that $A = \pm\infty$ corresponds to an executive biasing the result "as much as possible." If there are any outside constraints from, for example, the legal system, the executive will just re-optimize by setting A the highest or lowest value in that constraint set. We make no claims about what that set contains, and hence, we allow to vary over the entire extended real line. A third justification is that, fixing one particular rule, an executive can effectively ensure that measured benefits either exceed costs or are lower than costs by setting A to some very high or very low value. In our case, given that $\widetilde{B}$ is normally distributed and therefore has support over the real line, any finite value of A will feature positive approval and rejection probabilities. However, in practice, the executive could likely place some bounds on such that there exists a finite value of where the executive will be "sure enough" of approval/rejection; this is basically akin to them setting $A = \pm\infty$ in our model.

The marginal cost of increasing A is the increased welfare loss from type 2 errors and greater proposal costs.

We can immediately establish the executive's preferences for bias when $\alpha \geq \beta$.

Lemma OA6 *$A^* = \infty$ and $B^*_{sa} = \frac{C+k}{\beta}$ if and only if $\alpha \geq \beta$, where A^* is the optimal bias and $B^*_{sa} = B_{sa}(A^*)$.*

In light of Proposition OA5, an executive with welfare weight $\alpha \geq \beta$ would bias strict BCA upward as extremely as possible so that the executive could replicate the outcome with mandatory BCA with discretion. In this case, then, $EU^E_{sa} = EU^E_m$. Given Lemma OA3 we then have the following result.

Lemma OA7 *If $\alpha \geq \beta$ and $k_a > 0, EU^E_{sa} < EU^E_n$, i.e., an executive whose welfare weight is at least as large as the regulator's would prefer prohibiting BCA over mandating strict and biased BCA.*

The preferred level of bias clearly depends on the executive's welfare weight. From (40) and (41) it is clear that $MB(A, \alpha)$ increases in α for any given A, while $MC(A, \alpha)$ decreases in α. Thus, $\frac{\partial EU^E(A,\alpha)}{\partial A \partial \alpha} > 0$, i.e., $EU^E(\cdot)$ is strictly supermodular in (A, α). Strict Monotonicity Theorem 1 in Edlin and Shannon (1998) immediately implies the following.

Proposition OA9 *Suppose $\alpha' > \alpha$; $A^*(\alpha) \in \arg\max_{A \in \overline{\mathbb{R}}} EU^E_{sa}(A, \alpha)$; $A^*(\alpha') \in \arg\max_{A \in \overline{\mathbb{R}}} EU^E_{sa}(A, \alpha')$; and $A^*(\alpha) < \infty$. Then, $A^*(\alpha) < A^*(\alpha')$, i.e., an executive with a higher welfare weight will choose to bias benefits upward more than an executive with a lower welfare weight.*

The intuition is straightforward. The higher its welfare weight, the more concerned the executive will be about reducing type 1 errors and the less concerned about type 2 errors and proposal costs. Increasing the bias trades off reductions in the former for increases in the latter. Proposition OA9 (and its underlying intuition) is consistent with the results in the two-type model in the paper.

4.6.1 Computational analysis

To further explore the executive's preferences for biasing BCA, we compute the optimal solution to the problem in (39) for each combination of parameters in Table OA3. Figure OA11 reflects the main insights from this computational analysis. The top panel shows the optimal bias A^* as a function of the executive's regulation-sympathy index $\widehat{F}_0\left(\frac{C}{\alpha}\right)$. (In the computations being summarized here, all parameters but α and σ_b are set at their baseline values.) Note that the place where each of the lines "start" reflect the first value of A^* greater than $-\infty$. (For example, focusing on the turquoise line with $\sigma_b = 100$, it "starts" at a regulation-sympathy index of about 0.07. This implies that when $\sigma_b = 100$ and $\widehat{F}_0\left(\frac{C}{\alpha}\right) < 0.07$, $A^* = -\infty$.)

Furthermore, the lines turn from solid to dashed whenever $EU_n^E > EU_{sa}^E$; in other words, when the executive would prefer to prohibit BCA entirely rather than institute strict BCA with its optimal level of bias.

Consistent with Lemma OA6, $A^*(\alpha) = \infty$ only for $\alpha \geq \beta$. (Recall that in the baseline parameterization, $\beta = 1$, implying that $\alpha \geq \beta$ when $\widehat{F}_0(\frac{C}{\alpha}) \geq 0.933$.) Consistent with Proposition OA9, $A^*(\alpha)$ is strictly increasing in α. These panels illustrate that as long as BCA provides a somewhat informative signal ($\sigma_b <$ 1000), even when the regulator is only somewhat more regulation averse than the regulator, the optimal bias is typically negative, i.e., the executive benefits from the use of methodologies that undercount the net social benefits of proposed regulations. And indeed, as the executive becomes more regulation averse, this negative bias tends to accelerate, and eventually it "falls off a cliff" and becomes infinitely negative. This pattern suggests an asymmetry in the preferred degree of bias implemented by executive administrations. For two administrations with moderate levels of α, the preferred degree of bias does not differ much, even if one is regulation averse and the other regulation sympathetic. But those preferred degrees of bias might be very different from the highly negative bias preferred by a very regulation-averse administration. For example, if $\sigma_b = 100$ (our baseline value) a center-left administration with a regulation-sympathy index of 0.75 would prefer a BCA methodology with a slight negative bias, while a center-right administration with a regulation-sympathy index of 0.4 would prefer a moderately negatively biased methodology. These two administrations would utilize different BCA methodologies—perhaps the center-left administration might attempt to include co-benefits in their BCA analyses more so than the center-right administration—but their analytical frameworks would broadly resemble each other. But an administration that is highly regulation averse, with an index of 0.05, would prefer a methodology that is so biased that it would be guaranteed to reject any proposal coming from a regulatory agency. This administration's approach to BCA would look very different from either of the previous two. This insight is consistent with our analytical findings in the two-type model.

The top panel of Figure OA11 also shows that a less precise BCA exaggerates the effect of regulation sympathy on optimal bias. As σ_b increases, regulation-sympathetic executives tend to prefer more positive bias and regulation-averse executives tend to prefer more negative bias, with the crossover point at around a regulation-sympathy index of 0.45. Strikingly, if BCA is extremely noisy ($\sigma_b = 1,000$), any regulation-averse executive will prefer to bias benefits infinitely downward, while the bias of a regulation-neutral or regulation-sympathetic executive shoots up towards ∞ very quickly. We expect to see some of this exaggeration because for noisier BCA, the executive needs to bias BCA more to achieve its desired effect on the approval probability.

The second panel normalizes the marginal effect of a unit of bias by plotting the executive's prior probability of approving a regulation when using strict and biased BCA, $\int_{B_{sa}^*}^{\infty} \widehat{\Psi}\left(\frac{C-A^*-x}{\sigma_b}\right) f_0(x)dx$. Increasing this metric by one percentage point directly corresponds to a one percentage point increase in the probability the executive approves a rule. When $A^* = -\infty$, this value is zero, since the executive will not approve any regulations. However, when $A^* = \infty$, the value does not reach one; instead, it is $\widehat{F}_0\left(\frac{C+k}{\beta}\right)$, or the regulation-sympathy index of the regulator (which doubles as the lowest possible pivotal benefit for the regulator). This panel illustrates that as BCA gets more precise, the approval probability gets closer and closer to the 45-degree line. Intuitively, this makes sense; the executive's regulation-sympathy index is by construction the proportion of rules it would like to approve, and with a very precise BCA, the executive can very precisely tailor the level of bias to nearly eliminate both type 1 and type 2 errors and move the *ex ante* approval probability to its preferred level. However, a noisy BCA prevents the executive from doing so, and makes the executive rely more on its priors. This effect culminates when $\sigma_b = 1000$, when the executive approves effectively all the rules it can when it is regulation sympathetic, and rejects any rule when it is regulation averse.

The bottom panel in Figure OA11 shows the value to the executive of strict BCA with bias over a discretionary BCA mandate, $EU_{sa}^E - EU_m^E$. Again, we see a strong asymmetry. For even a moderately regulation-averse executive, the value from using biased BCA methodologies is modest, and as the executive becomes more regulation sympathetic, it approaches zero. Bias is most valuable for strongly regulation-averse executives. For example, for our baseline parameterization, $EU_{sa}^E - EU_m^E$ attains its peak around a regulation-sympathy index of 0.05. For more regulation-sympathetic executives, bias can still be somewhat valuable if BCA is very accurate, but its value to the executive is much lower. Negatively biased methodologies are most valuable for regulation-averse executives that are just on the cusp of the "bias cliff" where $A^*(\alpha)$ rapidly falls off to $-\infty$.

Further, Figure OA11 shows that the value of biased methodologies can be greatest for the executive when the BCA signal is very noisy. Recall that a regulation-averse executive is particularly vulnerable to a type 2 error because its preferred approval threshold, $\frac{C}{\alpha}$, is very high, above the mean of the prior distribution. The executive's vulnerability to type 2 errors with unbiased BCA is particularly high when σ_b is high because the noisy BCA increases the likelihood of BCA "draws" that exceed the approval threshold even when the underlying benefit does not exceed $\frac{C}{\alpha}$. A negatively biased BCA helps the executive reduce or eliminate the cost of type 2 errors.

Summarizing our computational findings:

Result 11 *The optimal bias $A^*(\alpha)$ can be positive or negative. It tends to be negative for regulation-averse executives. There is an asymmetry in the optimal bias across the spectrum of regulation sympathy. While the optimal level of bias does not differ much between moderately regulation-averse and moderately regulation-sympathetic executives, it differs drastically between moderately and strongly regulation-averse executives.*

Result 12 *The value* $EU^E_{sa} - EU^E_m$ *of a strict BCA standard with bias is greatest for a strongly (but not overwhelmingly) regulation-averse executive. That value tends to be maximal for an executive that prefers an infinitely negative bias, but is on the cusp of preferring a finite level of bias. The value of bias to those executives who value it the most tends to be higher as BCA becomes noisier.*

4.7 Summing up

With the exception of the non-existence of a maximally separating equilibrium under voluntary BCA in the two-type model, our results in the two-type model have counterparts—either analytically or computationally—in the continuous-type model with normal priors and BCA signals. And even with voluntary BCA in the continuous-type model, a maximally separating equilibrium existed for a narrow swath of parameter space when (as we would expect) k_a is small in comparison to k_r. Moreover, voluntary BCA was rarely optimal for the executive.

Assumptions 1 and 2 in the two-type model do limit the degree of alignment between the executive and regulator, something which our continuous-type model relaxes. That is, our continuous-type model allows for both a "big" agency problem between the executive and the regulator and a "small one," whereas the two-type model considers only a "big" agency problem. However, it is not difficult to extend the two-type model to allow closer alignment between the executive and the regulator, and when we do so, we find results that correspond to the continuous-type model: only a high-benefit regulator would propose; the executive approves a new regulation irrespective of the outcome of the BCA, and the executive is either indifferent between mandating or prohibiting BCA ($k_a = 0$) or strictly prefers prohibiting BCA ($k_a > 0$).

All of this suggests to us that there little loss of generality from the analysis of the two-type model.

5 Discretionary BCA with asymmetric bias

In the paper, we assume that bias changed the mean of the measured benefit. However, it is also possible that the executive could adopt methodologies that changed the information structure of the benefit-cost signal. For example, by mandating that the BCA methodology utilize a very high discount rate, the executive can ensure that the analysis will be very informative about a regulation that ends up with a high measured benefit (since the high discount rate would make this outcome difficult to achieve unless the underlying social benefit was quite large), but it will not be as informative about a proposed regulation that ends up with a low measured benefit (because the high discount rate predisposes the analysis in this direction).

To model this, we return to the two-type model in the paper. Our analysis focuses on the case of misalignment between the executive and regulator, which encompasses both Assumptions 2 and 3 in the paper. We also, in this section, continue to allow for the possibility of non-negative direct cost of a BCA, i.e., $k_a \geq 0$. Throughout this section, then, we assume $\beta \geq \frac{C+k}{B^L}$, and we let $\beta_m \equiv \frac{C}{B^L} + \frac{k}{(1-q)B^L}$.

We assume the executive commits to BCA methodologies such that the probability of a high measured benefit is changed by an amount x, i.e., $\Pr(\widetilde{b} = b^H | \widetilde{B} = B^H) = q + x$ and $\Pr(\widetilde{b} = b^H | \widetilde{B} = B^L) = 1 - q + x$, where $x \in [-(1-q), 1-q]$. This approach creates an asymmetry in the informativeness of $\widetilde{b}$. Because $\Pr(\widetilde{b} = b^L | \widetilde{B} = B^L) = q - x$, then if $x > 0$ BCA signal is more likely to identify the "correct" underlying state when $\widetilde{B} = B^H$ than when $\widetilde{B} = B^L$. If $x < 0$, the reverse is true. In this model of bias, we assume that the executive has discretion in how to use the results of the BCA in its approval decision.

Before stating the equilibrium, it is useful to see how the bias x changes key expressions.

$$E[\widetilde{B}|\widetilde{b} = b^H, x] = \frac{p(q+x)}{p(q+x) + (1-p)(1-q+x)} B^H + \frac{(1-p)(1-q+x)}{p(q+x) + (1-p)(1-q+x)} B^L. \tag{42}$$

$$E[\widetilde{B}|\widetilde{b} = b^L, x] = \frac{p(1-q-x)}{p(1-q-x) + (1-p)(q-x)} B^H + \frac{(1-p)(q-x)}{p(1-q-x) + (1-p)(q-x)} B^L. \tag{43}$$

It is straightforward to show that $E[\widetilde{B}|\widetilde{b} = b^H, x] > E[\widetilde{B}|\widetilde{b} = b^L, x]$ for all $x \in [-(1-q), 1-q)]$; $\frac{\partial E[\widetilde{B}|\widetilde{b}=b^H,x]}{\partial x} < 0$, $\frac{\partial E[\widetilde{B}|\widetilde{b}=b^L,x]}{\partial x} < 0$; and

$$B^H \leq E[\widetilde{B}|\widetilde{b} = b^H, x] \leq \frac{pB^H + 2(1-q)(1-p)B^L}{p + 2(1-q)(1-p)} \text{ for } -(1-q) \leq x < 1-q.$$

$$\frac{2(1-q)pB^H + (1-p)B^L}{2(1-q)p + 1 - p} \leq E[\widetilde{B}|\widetilde{b} = b^L, x] \leq B^L \text{ for } -(1-q) \leq x < 1-q.$$

To illustrate these inequalities, suppose that the executive sets $x = 1 - q$, the maximal upward distortion. Then, $\Pr(\widetilde{b} = b^H | \widetilde{B} = B^H) = 1$ and $\Pr(\widetilde{b} = b^H | \widetilde{B} = B^L) = 2(1-q)$. Thus, a high measured benefit could be consistent with either a low or a high underlying social benefit, but a low measured benefit could only arise if the true social benefit was low. The posterior expectation in the latter case is thus B^L. By contrast if $x = -(1-q)$, $\Pr(\widetilde{b} = b^H | \widetilde{B} = B^H) = 2q - 1$ and $\Pr(\widetilde{b} = b^H | \widetilde{B} = B^L) = 0$, so a high measured benefit is only consistent with a high underlying social benefit—so the posterior expectation is B^H—while a low measured benefit is consistent with both underlying states.

More generally, the higher is x, the more likely it is that the measured benefit will be high. Consequently, when it updates its beliefs upon seeing a measured benefit, the executive downweights the possibility that the underlying state is B^H and upweights the chance that the state is B^L. This is why the posterior expectations are decreasing in x. We can think of the executive as partially de-biasing its beliefs in light of its commitment to x. Indeed, increasing x can be thought of as structuring a BCA methodology that does a better job of identifying the worst case scenario. An example is the choice of a discount rate. By committing to the use of a low discount rate in discounting future benefits and costs, the executive biases the analysis so there is a higher probability that the measured benefit is high. However, in interpreting the data *ex post*, a BCA study that results in low measured net social benefit when a low discount rate is used is very bad news, while one that results in a high measured benefit is possibly good news, but not as good as it would have been if the measured benefit had been positive with a high discount rate.

The equilibrium (as a function of x and denoted by the subscript a) can be derived in the same way that we derived the equilibrium in the paper. When there is sufficient alignment between the executive and the regulator, i.e., $\beta < \frac{C+k}{B^L}$, the choice of x does not change the equilibrium outcome or the executive's *ex ante* welfare: in this equilibrium, $\rho_a^* = \rho_m^* = (1,0)$ and $\phi_A^* = \phi_m^* = (1,1)$. When $\beta \geq \frac{C+k}{B^L}$, there exists an equilibrium in the regulatory proposal subgame with mandatory BCA with asymmetric bias and discretion such that $\rho_a^* \neq (0,0)$. Let $\overline{\alpha}_m(b^H,x) = \frac{C}{E[\widetilde{B}|\widetilde{b}=b^H,x]}$, $\overline{\alpha}_m(b^L,x) = \frac{C}{E[\widetilde{B}|\widetilde{b}=b^L,x]}$, and $\overline{\beta}_m(x) \equiv \frac{C}{B^L} + \frac{k}{(1-q+x)B^L}$.

1. If $\alpha \geq \overline{\alpha}_m(b^L,x)$, then the equilibrium is $\rho_a^* = (1,1)$, $\phi_a^* = (1,1)$.

2. If $\alpha \leq \overline{\alpha}_m(b^L,x)$ and $\beta \leq \overline{\beta}_m(x)$, then the equilibrium is $\rho_a^* = (1, \rho_a^*(B^L))$, $\phi_a^* = (1, \phi_a^*(b^L))$, where

$$\rho_a^*(B^L) = \frac{p(1-q-x)(\alpha B^H - C)}{(1-p)(q-x)(C-\alpha B^L)} \in (0,1],$$
$$\phi_a^*(b^L) = \frac{k}{(q-x)(\beta B^L - C)} - \frac{1-q+x}{q-x} \in (0,1].$$

3. If $\alpha \in [\overline{\alpha}_m(b^H,x), \overline{\alpha}_m(b^L,x)]$ and $\beta \geq \overline{\beta}_m(x)$, then the equilibrium is $\rho_a^* = (1,1)$, $\phi_a^* = (1,0)$.

4. If $\alpha \leq \overline{\alpha}_m(b^H,x)$ and $\beta \geq \overline{\beta}_m(x)$, then the equilibrium is $\rho_a^* = (1, \rho_a^*(B^L))$, $\phi_a^* = (\phi_a^*(b^H), 0)$, with

$$\rho_a^*(B^L) = \frac{p(q+x)(\alpha B^H - C)}{(1-p)(1-q+x)(C-\alpha B^L)} \in (0,1],$$
$$\phi_a^*(b^H) = \frac{k}{(1-q+x)(\beta B^L - C)} \in (0,1].$$

The executive's *ex ante* welfare is

$$EU_a^E(x) = \begin{cases} p(\alpha B^H - C - k) & \beta \leq \frac{C+k}{B^L} \\ \alpha(pB^H + (1-p)B^L) - C - k & \beta \in \left(\frac{C+k}{B^L}, \overline{\beta}_m(x)\right], \ \alpha > \overline{\alpha}_m(b^L,x) \\ p\left(\left(\frac{2q-1}{q-x}\right)(\alpha B^H - C) - \left(1 + \frac{(1-q-x)(\alpha B^H - C)}{(q-x)(C-\alpha B^L)}\right)k\right) & \beta \in \left(\frac{C+k}{B^L}, \overline{\beta}_m(x)\right], \ \alpha < \overline{\alpha}_m(b^L,x) \\ p(q+x)(\alpha B^H - C) - (1-p)(1-q+x)(C-\alpha B^L) - k & \beta > \overline{\beta}_m(x), \ \alpha \in [\overline{\alpha}_m(b^H,x), \overline{\alpha}_m(b^L,x)] \\ -kp\left(1 + \frac{(q+x)(\alpha B^H - C)}{(1-q+x)(C-\alpha B^L)}\right) & \beta > \overline{\beta}_m(x), \alpha < \overline{\alpha}_m(b^H,x) \end{cases} \quad (44)$$

Define $\overline{x}_m(\beta)$ as the inverse of $\overline{\beta}_m(x)$, i.e., $\overline{x}_m(\beta) = \frac{k}{\beta B^L - C} - (1-q)$. Note that $EU_a^E(x)$ is discontinuous at $\overline{x}_m(\beta)$. However straightforward algebra establishes that $EU_a^E(x)$ is continuous in x both for $x > \overline{x}_m(\beta)$ and $x < \overline{x}_m(\beta)$.

We can notice some interesting features of the equilibrium. First, when $x = 1-q$, then $\overline{\alpha}_m(b^L, 1-q) = \frac{C}{E[\widetilde{B}|\widetilde{b}=b^L,1-q]} = \frac{C}{B^L}$ and $\overline{\beta}_m(1-q) = \frac{C}{B^L} + \frac{k}{2(1-q)B^L}$. It follows that for $\beta \in (\frac{C+k}{B^L}, \frac{C}{B^L} + \frac{k}{2(1-q)B^L})$ and for all $\alpha \in \left(\frac{C}{B^H}, \frac{C}{B^L}\right)$, $\rho_a^*(B^L) = 0$. Thus when there is just modest misalignment, the maximum asymmetric bias can enable the executive to replicate the full-information solution. That is, $x = 1-q$ can create effective alignment in cases where we would have had misaligned BCA in the absence of bias. The intuition is that as x gets closer to $1-q$, a measured benefit b^L becomes an extremely strong signal that the underlying state is B^L, and it becomes a perfect signal when $x = 1-q$. Given this, even a small proposal probability by a type-B^L regulator, leads the executive to believe that $\alpha E[\widetilde{B}|\widetilde{b} = b^L, x] < C$, and consequently, the executive will optimally set $\phi_a(b^L) = 0$ for almost any conjectured $\rho_a(B^L)$. A type-B^L regulator would then expect that if it did propose, the expected probability of approval would be $E\left[\phi_a|B^L, x\right] = (1-q+x)\,\phi_a(b^H)$. As long as there is modest misalignment, $E\left[\phi_a|B^L, x\right]\left[\beta B^L - C\right] - k < 0$, even if $\phi_a(b^H) = 1$. Thus, by maximally biasing the BCA signal, the executive can choke off the proposal of a rule with low net social benefits, *provided that there is only modest misalignment.*

Second, maximally biasing the BCA signal upward is not sufficient to deter weak proposals when there is severe misalignment, i.e., $\beta > \overline{\beta}_m(x)$: even a type-$B^L$ will be highly motivated to propose in this situation. And this creates a trade-off for the executive because increasing x decreases $\overline{\beta}_m(x)$ and thus expands the range in which we have severe misalignment. In fact, we can show that $EU_a^E(x)$ jumps discontinuously downward at the value of x that tips the executive from the case of modest misalignment to that of severe misalignment.

The executive's optimal bias solves the problem

$$\max_{x \in [-(1-q),(1-q)]} EU_a^E(x).$$

Let $x^*(\alpha, \beta) = \arg\max_{x\in[-(1-q),(1-q)]} EU_a^E(x)$ be the (possibly non-unique) solution to this problem.

Proposition OA10 *Suppose the regulator and executive are misaligned, i.e.,* $\beta > \frac{C+k}{B^L}$. *(a) If* $\beta \in \left(\frac{C+k}{B^L}, \frac{C}{B^L} + \frac{k}{2(1-q)B^L}\right)$, *for all* $\alpha \in \left(\frac{C}{B^H}, \frac{C}{B^L}\right)$, $x^*(\alpha,\beta) = 1-q$ *is the unique solution to the executive's problem; (b) If* $\beta > \frac{C}{B^L} + \frac{k}{2(1-q)B^L}$ *and* $\alpha \in \left(\frac{C}{B^H}, \overline{\alpha}_n\right)$, *then* $x^*(\alpha,\beta) = \overline{x}_m(\beta)$ *is the unique solution to the executive's problem; (c) If* $\beta > \frac{C}{B^L} + \frac{k}{2(1-q)B^L}$ *and* $\alpha = \overline{\alpha}_n$, *then* $x^*(\alpha,\beta) = \overline{x}_m(\beta)$ *is the unique solution to the executive's problem. (d) If* $\beta > \frac{C}{B^L} + \frac{k}{2(1-q)B^L}$ *and* $\alpha \in \left(\overline{\alpha}_n, \overline{\alpha}_m(b^L, \overline{x}_m(\beta))\right)$, $x^*(\alpha,\beta) = \overline{x}_m(\beta)$ *or* $1-q$ *is a solution to the executive's problem, depending on whether* $p\left(\left(\frac{2q-1}{q-\overline{x}_m(\beta)}\right)(\alpha B^H - C) - \left(1 + \frac{(1-q-\overline{x}_m(\beta))(\alpha B^H - C)}{(q-\overline{x}_m(\beta))(C-\alpha B^L)}\right)k\right) \gtrless$

$p(\alpha B^H - C) - (1-p)2(1-q)(C - \alpha B^L) - k$. *If* $\alpha \in \left(\overline{\alpha}_m(b^L, \overline{x}_m(\beta)), \frac{C}{B^L}\right)$, $x^*(\alpha,\beta) = 1-q$ *is the unique solution to the executive's problem.*

This result implies that a regulation-sympathetic executive's optimal bias is at least as large as that of a regulation-averse or regulation-neutral executive.

A direct implication of Proposition OA10 is that when there is severe misalignment, a regulation-neutral or regulation-averse executive may prefer to bias the BCA downward.

Corollary 1 *If* $\beta > \overline{\beta}_m$ *and* $\alpha \in \left(\frac{C}{B^H}, \overline{\alpha}_n\right]$ *then* $x^*(\alpha,\beta) = \overline{x}_m(\beta) < 0$ *is the unique solution to the executive's problem, i.e., when there is severe misalignment and the executive is regulation averse or regulation neutral, the executive prefers to bias the BCA signal downward.*

6 Full commitment model

The full commitment solution, denoted by subscript f solves

$$\max_{\rho_f(B^H),\rho_f(B^L),\phi_f(b^H),\phi_f(b^H)} p\left(\rho_f(B^H)\left\{E[\phi_f(\widetilde{b})|B^H](\alpha B^H - C) - k\right\}\right),$$
$$+ (1-p)\left(\rho_f(B^L)\left\{E[\phi_f(\widetilde{b})|B^L](\alpha B^L - C) - k\right\}\right)$$

subject to:

$$\rho_f(B^H) = \begin{cases} 1 & E[\phi_f(\widetilde{b})|B^H](\beta B^H - C) - k > 0 \\ \in [0,1] & E[\phi_f(\widetilde{b})|B^H](\beta B^H - C) - k = 0 \\ 0 & E[\phi_f(\widetilde{b})|B^H](\beta B^H - C) - k < 0 \end{cases}. \tag{45}$$

$$\rho_f(B^L) = \begin{cases} 1 & E[\phi_f(\widetilde{b})|B^L](\beta B^L - C) - k > 0 \\ \in [0,1] & E[\phi_f(\widetilde{b})|B^L](\beta B^L - C) - k = 0 \\ 0 & E[\phi_f(\widetilde{b})|B^L](\beta B^L - C) - k < 0 \end{cases}. \tag{46}$$

$\phi_f(b^H) \in [0,1], \phi_f(b^H) \in [0,1]$,

where

$$E[\phi_f(\widetilde{b})|B^H] = q\phi_f(b^H) + (1-q)\phi_f(b^L).$$
$$E[\phi_f(\widetilde{b})|B^L] = (1-q)\phi_f(b^H) + q\phi_f(b^L).$$

We characterize the solution to this problem for all β: $\beta \le \frac{C+k}{B^L}$, $\beta \in \left(\frac{C+k}{B^L}, \overline{\beta}_m\right)$, and $\beta \ge \overline{\beta}_m$, where, as in the previous section, $\overline{\beta}_m \equiv \frac{C}{B^L} + \frac{k}{(1-q)B^L}$. Of course, only this last case is considered in the main paper. We also allow for a non-negative direct cost of BCA, k_a, with $k = k_a + k_r$ and $k \geq k_r$.

Proposition OA11 *The solution to the executive's problem under full commitment is as follows. (1) If* $\alpha \geq \frac{C}{B^H} + \frac{k}{E[\phi_f^*(\tilde{b})|B^H]B^H}$

$$\rho_f^*(B^H) = 1.$$

$$\rho_f^*(B^L) = 0.$$

$$\phi_f^*(b^H) = \begin{cases} 1 & \beta \leq \frac{C+k}{B^L} \\ 1 & \beta \in \left(\frac{C+k}{B^L}, \overline{\beta}_m\right) \\ \frac{k}{(1-q)(\beta B^L - C)} & \beta \geq \overline{\beta}_m \end{cases} . \tag{47}$$

$$\phi_f^*(b^L) = \begin{cases} 1 & \beta \leq \frac{C+k}{B^L} \\ \frac{k}{q(\beta B^L - C)} - \frac{(1-q)}{q} & \beta \in \left(\frac{C+k}{B^L}, \overline{\beta}_m\right) \\ 0 & \beta \geq \overline{\beta}_m \end{cases} . \tag{48}$$

(2) If $\alpha < \frac{C}{B^H} + \frac{k}{E[\phi_f^*(\tilde{b})|B^H]B^H}$, $\rho_f^*(B^H) = \rho_f^*(B^L) = \phi_f^*(b^H) = \phi_f^*(b^L) = 0.$

In all cases, the executive's full-commitment solution chokes off proposals from a low-type regulator. By contrast, in Proposition 2 in the paper, under mandatory BCA there is always a positive proposal probability from a low-benefit regulator. (As discussed in Section 3.4 of the paper, there is also a positive proposal probability from a low-benefit regulator in the equilibrium when we replace Assumption 2 in the paper with Assumption 3). Thus, the equilibrium in the mandatory BCA subgame does not implement the full-commitment outcome.[24]

Does the equilibrium under strict and biased BCA presented in the paper implement the full-commitment solution characterized in Proposition OA11? In general, no. When $\alpha \geq \frac{C}{B^H} + \frac{k}{E[\phi_f^*(\tilde{b})|B^H]B^H}$ and $\beta \geq \overline{\beta}_m$ (i.e., Assumption 2 in the paper is satisfied), (47) implies that the full-commitment solution is not a strict benefit-cost standard: it is optimal for the executive to commit to rejecting some proposals that have a high benefit-cost signal. This means that for that range of α for which the $A^*(\alpha) = 0$, the outcomes under strict BCA with bias and the full commitment must differ (since $A^*(\alpha) = 0$ implies that the executive is utilizing a strict benefit-cost standard). Is $A^*(\alpha) = 0$ consistent with the condition $\alpha \geq \frac{C}{B^H} + \frac{k}{E[\phi_f^*(\tilde{b})|B^H]B^H} = \frac{C}{B^H} + \frac{(1-q)\left(\beta B^L - C\right)}{qB^H}$? There are parameterizations for which the answer is yes. For example, if $B^H = 700, B^L = 200, C = 100, p = 0.5, q = 0.75$, $k_r = 4, k_a = 0$, and $\beta = 2$, then Assumption 2 is satisfied (since $\overline{\beta}_m = 1.66$). We have $\overline{\alpha}_m(b^H) = 0.273$, $\overline{\alpha}_n = 0.375$, and $\overline{\alpha}_m(b^L) = 0.600$. In addition, the values of α for which $\phi_f^*(b^H) = \frac{k}{(1-q)(\beta B^L - C)} = 0.32$ and $\phi_f^*(b^L) = 0$ are $\alpha \geq 0.238$. Further, it can be verified that the

[24] However, if $\beta \leq \frac{C+k}{B^L}$, a case ruled out by Assumption 2 in the paper but analyzed earlier in Section 2 above, the equilibrium in the mandatory BCA subgame does coincide with the full-commitment solution.

sufficient condition for Proposition 7 in the paper holds. In this case, $\widehat{\alpha}_s = 0.287$ and thus $A^*(\alpha) = 0$ for $\alpha \in [0.287, 0.600]$. For any α in this range, the executive would adopt a strict BCA standard with no bias, while the full-commitment solution is "tougher" than a strict BCA standard.

There is a case where strict BCA with bias would implement the full-commitment solution. For α sufficiently close to zero, the full-commitment solution is to reject any proposal, and this would correspond to the the case of strict BCA with downward bias.

7 Proofs

Proof of Lemma OA1:
Suppose, to the contrary, that we had an equilibrium in which for $B' \neq B''$, $\phi' \neq \phi''$. Then necessarily

$$\begin{aligned} \phi' [\beta B' - C] - k &\geq \phi'' [\beta B' - C] - k. \\ \phi'' [\beta B'' - C] - k &\geq \phi' [\beta B'' - C] - k. \\ \phi' [\beta B' - C] - k &\geq 0. \\ \phi'' [\beta B'' - C] - k &\geq 0. \end{aligned}$$

The latter two conditions imply $\beta B' - C > 0$ and $\beta B'' - C > 0$. The first two inequalities then imply $\phi' \geq \phi''$ and $\phi' \leq \phi''$, respectively, which means that $\phi' = \phi''$, contradicting the contrapositive assumption.■

Proof of Proposition OA1. First, we establish that the proposed equilibrium in indeed an equilibrium. With $\phi_n^* = 1$, $\frac{C}{\beta} + \frac{k_r}{\beta \phi_n^*} = \frac{C+k_r}{\beta} \in (B^L, B^H)$, where the set inclusion is equivalent to our maintained assumption. Therefore, $\rho_n(\phi_n^*) = (1, 0) = \rho_n^*$. Further, $E[\widetilde{B}|\rho_n^*] = B^H > \frac{C}{\alpha}$, where the last inequality follows from Assumption 1 in the paper. Therefore, $\phi_n(\rho_n^*) = 1 = \phi_n^*$.

We next show that this equilibrium is unique using our solution concept, which incorporates the D1 refinement. First, we can eliminate any equilibria where $\rho_n(B^L) > 0$ because, when $\widetilde{B} = B^L$, alignment implies that proposing with positive probability is strictly dominated by not proposing at all for the regulator. Therefore, $\rho_n(\phi_n, B^L) = 0$ for all $\phi_n \in [0, 1]$.

We can then break into two cases. First, suppose that $\rho_n(\phi_n, B^H) > 0$. Then $E[\widetilde{B}|\rho_n] = B^H$. Assumption 1 then gives us that $\phi_n(\rho_n) = 1$.

Second, suppose $\rho_n(\phi_n, B^H) = 0$. Using the logic of footnote 14 in the paper, such equilibria do not survive D1, as the type of regulator that benefits most from proposing is the high type, and the executive would like to approve high-type regulations per Assumption 1.■

Proof of Proposition OA2. The proof is identical to the proof of Proposition OA1, but with k replacing k_r.■

Proof of Proposition OA3.We state necessary and sufficient conditions for each kind of equilibrium below, and show that they are mutually exclusive (except for when equalities between certain parameters hold), and they cover the entire parameter space.

First, $\rho_m^* = (1, 1)$, $\phi_m^* = (1, 1)$ is an equilibrium if and only if

$$E[\widetilde{B}|(1, 1), b^L] \geq \frac{C}{\alpha}. \tag{49}$$

Note that (49) implies that $E[\widetilde{B}|(1, 1), b^H] \geq \frac{C}{\alpha}$, and that our assumptions on the parameters imply that the regulator is responding optimally in this case, i.e. $B^L > \frac{C+k}{\beta}$. Rearranging (49) gives us the α cutoff in part (1) of the proposition.

Second, $\rho_m^* = (1, \rho_m^*(B^L))$, $\phi_m^* = (1, \phi_m^*(b^L))$ with $\rho_m^*(B^L), \phi_m^*(b^L) \in (0,1]$ is an equilibrium if and only if

$$\frac{C}{\beta} + \frac{k}{\beta(1-q+q\phi_m^*(b^L))} = B^L, \tag{50}$$

$$\frac{p(1-q)}{p(1-q)+q(1-p)\rho_m^*(B^L)}B^H + \frac{q(1-p)\rho_m^*(B^L)}{p(1-q)+q(1-p)\rho_m^*(B^L)} = \frac{C}{\alpha}. \tag{51}$$

Notice that $\phi_m^*(b^L) \geq 0$ and $\rho_m^*(B^L) \leq 1$, along with our assumptions, imply that whenever these statements hold, we have $E[\widetilde{B}|(1,1), b^L] < \frac{C}{\alpha}$ and $\frac{C}{\beta} + \frac{k}{\beta(1-q)} \geq B^L$. Further, because $\lim_{\rho_m(B^L)\to 0} E[\widetilde{B}|(1, \rho_m(B^L)), b^L] = B^H > \frac{C}{\alpha}$ and $\lim_{\phi_m(b^L)\to 1} \frac{C}{\beta} + \frac{k}{\beta(1-q+q\phi_m(b^L))} = \frac{C+k}{\beta} < B^L$ by our assumptions, there will exist values of $\phi_m^*(b^L) < 1$ and $\rho_m^*(B^L) > 0$ that make (50) and (51) hold. Solving for these values in (50) and (51) gives us the α and β cutoffs in part (2).

Third, $\rho_m^* = (1,1)$, $\phi_m^* = (1,0)$ is an equilibrium if and only if

$$\frac{C}{\beta} + \frac{k}{\beta(1-q)} \leq B^L, \tag{52}$$

$$\frac{p(1-q)}{p(1-q)+q(1-p)}B^H + \frac{q(1-p)}{p(1-q)+q(1-p)}B^L \leq \frac{C}{\alpha}, \tag{53}$$

$$\frac{pq}{pq+(1-p)(1-q)}B^H + \frac{(1-p)(1-q)}{pq+(1-p)(1-q)}B^L \geq \frac{C}{\alpha}. \tag{54}$$

Rearranging (52)-(54) give us the α and β cutoffs in part (3).

Fourth and finally, $\rho_m^* = (1, \rho_m^*(B^L))$, $\phi_m^* = (\phi_m^*(b^H), 0)$ with $\rho_m^*(B^L), \phi_m^*(b^H) \in (0,1]$ is an equilibrium if and only if

$$\frac{C}{\beta} + \frac{k}{\beta(1-q)\phi_m^*(b^H)} = B^L, \tag{55}$$

$$\frac{pq}{pq+(1-p)(1-q)\rho_m^*(B^L)}B^H + \frac{(1-p)(1-q)\rho_m^*(B^L)}{pq+(1-p)(1-q)\rho_m^*(B^L)}B^L = \frac{C}{\alpha}. \tag{56}$$

Notice that $\phi_m^*(b^H) \leq 1$ and $\rho_m^*(B^L) \leq 1$, along with our assumptions, imply that whenever (55) and (56) hold, we have $E[\widetilde{B}|(1,1), b^H] < \frac{C}{\alpha}$ and $\frac{C}{\beta} + \frac{k}{\beta(1-q)} \leq B^L$. Further, because

$$\lim_{\rho_m(B^L)\to 0} E[\widetilde{B}|(1, \rho_m(B^L)), b^H] = B^H > \frac{C}{\alpha},$$

$$\lim_{\phi_m(b^H)\to 0} \frac{C}{\beta} + \frac{k_r}{\beta(1-q)\phi_m(b^H)} = \infty > B^L,$$

there exist values of $\phi_m^*(b^H) > 0$ and $\rho_m^*(B^L) > 0$ that make these equalities hold. Solving for these values in (55) and (56) gives us the α and β cutoffs in part (4).

To conclude, unless $\alpha = E[\widetilde{B}|(1,1), b^L]$, $\alpha = E[\widetilde{B}|(1,1), b^H]$, or $\beta = \frac{C}{B^L} + \frac{k_r}{B^L(1-q)}$, then the parameter values are such that at most only one of these four sets of conditions hold. Because the union of these conditions is the entire parameter space, at least one of these four sets of conditions hold. Therefore, we have proven that unless $\alpha = E[\widetilde{B}|(1,1), b^L]$, $\alpha = E[\widetilde{B}|(1,1), b^H]$, or $\beta = \frac{C}{B^L} + \frac{k_r}{B^L(1-q)}$, there is exactly one equilibrium. ■ ■

Proof of Proposition OA4. The equilibrium occurs at the point (B_n, ϕ_n) at which the regulator's best response function $B_n(\phi_n)$ in (2) and the executive's best response function $\phi_n(B_n)$ in (3) are simultaneously satisfied. Inverting $B_n(\phi_n)$ gives us $B_n^{-1}(B_n) = \frac{k_r}{\beta B_n - C}$. Since the regulator will only propose a new rule if $B \geq \frac{C}{\beta} + \frac{k_r}{\beta\phi_n} \geq \frac{C+k_r}{\beta}$, the only pivotal benefits that are candidates for equilibrium are $B_n \in \left[\frac{C+k_r}{\beta}, \infty\right)$. Over this domain, $B_n^{-1}(B_n)$ is continuous, strictly decreasing, with $B_n^{-1}(\frac{C+k_r}{\beta}) = 1$ and $\lim_{B_n\to\infty} B_n^{-1}(B_n) = 0$.

For the executive's best response function in (3), we recall that $E\left[\widetilde{B}|\widetilde{B}\geq B_n\right] = B_0+\sigma_0 h\left(\frac{B_n-B_0}{\sigma_0}\right)$, and note that this is a continuous, strictly increasing function, with $\lim_{B_n\to-\infty} E\left[\widetilde{B}|\widetilde{B}\geq B_n\right] = B_0$, and $\lim_{B_n\to\infty} E\left[\widetilde{B}|\widetilde{B}\geq B_n\right] = \infty$. The executive's best response function is thus a continuous, multivalued function given by

$$\phi_n(B_n)=\begin{cases} 1 & \text{if } B_n \geq B_0+\sigma_0 h^{-1}\left(\frac{\frac{C}{\alpha}-B_0}{\sigma_0}\right) \\ \in[0,1] & \text{if } B_n = B_0+\sigma_0 h^{-1}\left(\frac{\frac{C}{\alpha}-B_0}{\sigma_0}\right) \\ 0 & \text{if } B_n \leq B_0+\sigma_0 h^{-1}\left(\frac{\frac{C}{\alpha}-B_0}{\sigma_0}\right), \end{cases}, \tag{57}$$

where $h^{-1}(\cdot)$ is the inverse standard normal hazard function.

The equilibrium occurs at a pivotal benefit at which $B_n^{-1}(B_n)-\phi_n(B_n)=0$. Given the properties of $B_n^{-1}(B_n)$ and $\phi_n(B_n)$, over the domain $B_n\in\left[\frac{C+k_r}{\beta},\infty\right)$, the function $B_n^{-1}(B_n)-\phi_n(B_n)$ is continuous and monotonically decreasing in B_n, mapping to all possible values in $[0,1]$, with a vertical portion at $B_n=B_0+\sigma_0 h^{-1}(\frac{\frac{C}{\alpha}-B_0}{\sigma_0})$. By the intermediate value theorem and the monotonicity of $B_n^{-1}(B_n)-\phi_n(B_n)$, there exists a unique B_n^* at which $B_n^{-1}(B_n^*)-\phi_n(B_n^*)=0$ and a unique ϕ_n^* given by $\phi_n^*=B_n^{-1}(B_n^*)\in(0,1]$. This establishes that an equilibrium exists and is unique.

Now, define $G(\alpha,\phi,\beta)\equiv E\left[\widetilde{B}|\widetilde{B}\geq\frac{C}{\beta}+\frac{k_r}{\beta\phi}\right]-\frac{C}{\alpha} = B_0+\sigma_0 h\left(\frac{\frac{C}{\beta}+\frac{k_r}{\beta\phi}-B_0}{\sigma_0}\right)-\frac{C}{\alpha}$. The function $G(\cdot)$ is continuous in its arguments; $\lim_{\alpha\to0}G(\alpha,\phi,\beta)=-\infty$; $\lim_{\phi\to0}G(\alpha,\phi,\beta)=\infty$, and $\frac{\partial G}{\partial\alpha}>0$, $\frac{\partial G}{\partial\phi}<0$, $\frac{\partial G}{\partial\beta}<0$. (The latter two inequalities follow because, as is well known, the standard normal hazard function is monotonically increasing; see Bagnoli and Bergstrom 2005). Moreover,

$$G(\beta,1,\beta)=E[\widetilde{B}|\widetilde{B}\geq\frac{C+k_r}{\beta}]-\frac{C}{\beta}\geq\frac{C+k_r}{\beta}-\frac{C}{\beta}>0. \tag{58}$$

Because $G(\cdot)$ is continuous in α, the intermediate value theorem implies that a solution for α to $G(\alpha,1,\beta)=0$ exists, and $\frac{\partial G}{\partial\alpha}>0$ implies that this solution, $\alpha_n^*(\beta)$, is unique and (given (58)) is strictly less than β. The solution $\alpha_n^*(\beta)$ can be written as

$$\alpha_n^*(\beta)=\frac{C}{B_0+\sigma_0 h\left(\frac{\frac{C+k_r}{\beta}-B_0}{\sigma_0}\right)}<\beta.$$

Note that

$$G(\alpha,1,\beta)<0 \text{ for } \alpha<\alpha_n^*(\beta). \tag{59}$$
$$G(\alpha,1,\beta)>0 \text{ for } \alpha>\alpha_n^*(\beta). \tag{60}$$

Also note that $G(\frac{C}{C+k_r}\beta,1,\beta)=E\left[\widetilde{B}|\tilde{B}\geq\frac{C+k_r}{\beta}\right]-\frac{C+k_r}{\beta}>0$, which implies that $\alpha_n^*(\beta)<\frac{C}{C+k_r}\beta$.

Now, suppose $\alpha<\alpha_n^*(\beta)<\frac{C}{C+k_r}\beta$. Because $G(\cdot)$ decreases in ϕ and $\lim_{\phi\to0}G(\alpha,\phi,\beta)=\infty$, the inequality in (59) and $\frac{\partial G}{\partial\phi}<0$ implies that for $\alpha<\alpha_n^*(\beta)$ there exists a unique $\phi_n^*\in(0,1)$ such that $G(\alpha,\phi_n^*,\beta)=0$, or equivalently, given the definition of $G(\cdot)$,

$$B_0+\sigma_0 h\left(\frac{\frac{C}{\beta}+\frac{k_r}{\beta\phi_n^*}-B_0}{\sigma_0}\right)=\frac{C}{\alpha}. \tag{61}$$

Now, let $B_n^*=\frac{C}{\beta}+\frac{k_r}{\beta\phi_n^*}$. Clearly (B_n^*,ϕ_n^*) satisfies the regulator's best response function $B_n=B_n(\phi_n)$. Because $\phi_n^*\in(0,1)$ and (61) implies, $B_n^*=\frac{C}{\beta}+\frac{k_r}{\beta\phi_n^*}=B_0+\sigma_0 h^{-1}\left(\frac{\frac{C}{\alpha}-B_0}{\sigma_0}\right)$, (B_n^*,ϕ_n^*) also satisfies the executive's best response condition, $\phi_n=\phi_n(B_n)$. Thus, for $\alpha<\alpha_n^*(\beta)$, conditions (4) and (5) characterize the equilibrium in the regulatory approval subgame when BCA is not used. We note that since $G(\alpha,\phi_n^*,\beta)=$

0, we have

$$\frac{d\phi_n^*}{\partial \alpha} = -\frac{\frac{\partial G}{\partial \alpha}}{\frac{\partial G}{\partial \phi}} > 0, \quad \frac{d\phi_n^*}{\partial \beta} = -\frac{\frac{\partial G}{\partial \beta}}{\frac{\partial G}{\partial \phi}} < 0.$$

Consider, finally, $\alpha \geq \alpha_n^*(\beta)$. Given (60) (along with $G(\alpha_n^*(\beta), 1, \beta) = 0$), it follows that $E[\widetilde{B}|\widetilde{B} \geq \frac{C+k_r}{\beta}] \geq \frac{C}{\alpha}$ for $\alpha \geq \alpha_n^*(\beta)$, which in turn implies that the executive's best response $\phi_n(B_n) = 1$ for all $B_n \in [\frac{C+k_r}{\beta}, \infty)$. That is, for any pivotal benefit in the relevant domain of the regulator, the executive will approve a proposed regulation with certainty. Note, too, that $B_n(1) = \frac{C+k_r}{\beta}$, so the pivotal benefit $B_n = \frac{C+k_r}{\beta}$ is a best reply to the approval probability $\phi_n^* = 1$. Thus, when $\alpha \geq \alpha_n^*(\beta)$, there is a pure strategy equilibrium given by $B_n^* = \frac{C+k_r}{\beta}$ and $\phi_n^* = 1$, establishing part (2) of the proposition. ■

Proof of Lemma OA2.
Preliminaries: We begin by stating some properties of the hazard function of the standard normal distribution $h(z)$: (a) $h'(z) > 0$ for $z \in (-\infty, \infty)$; (b) $h(z) > z$ for $z \in (-\infty, \infty)$; (c) $\lim_{z\to-\infty}(h(z) - z) = \infty$; (d) $\lim_{z\to\infty}(h(z) - z) = 0$; and (e) $h'(z) < 1$ for $z \in (-\infty, \infty)$. As noted in the proof of Proposition OA4, property (a) is well known. To establish property (b), it is straightforward to show that $h'(z) = h(z)(h(z) - z)$, which, given property (a) and the fact that $h(z) > 0$, implies that $h(z) > z$ for all $z \in (-\infty, \infty)$. For property (c), recall that $h(z) = \frac{\psi(z)}{\widehat{\Psi}(z)} = \frac{e^{-\frac{1}{2}z^2}}{\int_z^\infty e^{-\frac{1}{2}t^2}dt}$, so $\lim_{z\to-\infty} h(z) = \frac{\lim_{z\to-\infty} e^{-\frac{1}{2}z^2}}{\int_{-\infty}^\infty e^{-\frac{1}{2}t^2}dt} = \frac{0}{1}$, so it immediately follows that $\lim_{z\to-\infty}[h(z) - z] = \infty$.

To establish property (d), we can write $h(z) - z$ as

$$h(z) - z = \frac{e^{-\frac{1}{2}z^2} - z\int_z^\infty e^{-\frac{1}{2}t^2}dt}{\int_z^\infty e^{-\frac{1}{2}t^2}dt}.$$

Thus, we have

$$\begin{aligned}
\lim_{z\to\infty}[h(z) - z] &= \frac{\lim_{z\to\infty}\left[e^{-\frac{1}{2}z^2} - z\int_z^\infty e^{-\frac{1}{2}t^2}dt\right]}{\lim_{z\to\infty}\int_z^\infty e^{-\frac{1}{2}t^2}dt} = \frac{0}{0} \\
&= \frac{\lim_{z\to\infty}\int_z^\infty e^{-\frac{1}{2}t^2}dt}{\lim_{z\to\infty} e^{-\frac{1}{2}z^2}dt} = \frac{0}{0} \\
&= \frac{\lim_{z\to\infty} e^{-\frac{1}{2}z^2}}{\lim_{z\to\infty} ze^{-\frac{1}{2}z^2}} = \lim_{z\to\infty}\frac{1}{z} = 0,
\end{aligned}$$

where the equalities in the second and third lines follow from successively applying L'Hospital's rule.

To establish property (e), we begin by establishing that $h(z)$ is a strictly convex function. It is well known that the Mills ratio, $\frac{1}{h(z)}$, is convex in z (Baricz 2008). Now, $d\frac{\frac{1}{h(z)}}{dz} = \frac{h'(z)}{[h(z)]^2}$. Thus, $d^2\frac{\frac{1}{h(z)}}{dz^2} = \frac{h''(z)[h(z)]^2 - 2h(z)\left[h'(z)\right]^2}{h(z)^4} > 0$, so $h''(z) > \frac{2h(z)\left[h'(z)\right]^2}{[h(z)]^2} > 0$ for all $z \in (-\infty, \infty)$. Given that $h(z)$ is convex in z, to establish that $h'(z) < 1$ for all $z \in (-\infty, \infty)$, it suffices to show that $h'(z) < 1$ for $z \geq 0$, because the strict convexity of $h(\cdot)$ implies that $h'(z)$ is an increasing function of z. Now, according to Theorem 2.3 in Baricz (2008), for $z \geq 0$, $\frac{1}{h(z)} > \frac{2}{\sqrt{z^2+4}+z}$, so for $z \geq 0$, $h(z) < \frac{\sqrt{z^2+4}+z}{2}$. Since $h'(z) = h(z)(h(z) - z)$, $h(z) - z > 0$, and $\frac{\sqrt{z^2+4}+z}{2} > 0$ for $z \geq 0$, we have for $z \geq 0$

$$\begin{aligned}
h'(z) &< \left(\frac{\sqrt{z^2+4}+z}{2}\right)(h(z) - z) \\
&< \left(\frac{\sqrt{z^2+4}+z}{2}\right)\left(\frac{\sqrt{z^2+4}+z}{2} - z\right) \\
&= \frac{1}{4}\left(\sqrt{z^2+4}+z\right)\left(\sqrt{z^2+4}-z\right) = 1.
\end{aligned}$$

As indicated above, the convexity of $h'(z)$ then implies that $h'(z) < 1$ for all $z \in (-\infty, \infty)$. $\square$

Proof of lemma: Given the properties of $h(\cdot)$, we can write $h(z) = z + g(z)$, where $g(\cdot)$ is a continuous function with $g(z) > 0$, $g'(z) \in (-1, 0)$, $\lim_{z\to-\infty} g(z) = \infty$, and $\lim_{z\to\infty} g(z) = 0$. Thus, we can write the truncated expectation $E[\widetilde{B}|b, \widetilde{B} \geq B_m]$ as follows:

$$H(b, B_m) \equiv E[\widetilde{B}|b, \widetilde{B} \geq B_m] = B_m + \overline{\sigma}_m g\left(\frac{B_m - \overline{B}_m(b)}{\overline{\sigma}_m}\right).$$

Given (14) and the properties of $g(\cdot)$, we have $\lim_{b\to-\infty} H(b, B_m) = B_m$, $\lim_{b\to\infty} H(b, B_m) = \infty$, $\frac{\partial H(b,B_m)}{\partial b} = -g'\left(\frac{B_m - \overline{B}_m(b)}{\overline{\sigma}_m}\right)\frac{d\overline{B}_m(b)}{db} > 0$, and $\frac{\partial H(b,B_m)}{\partial B_m} = 1 + g'(\frac{B_m - \overline{B}_m(b)}{\overline{\sigma}_m}) > 0$.

When $B_m < \frac{C}{\alpha}$, because $H(\cdot)$ is a continuous function in b, the intermediate value theorem implies that there exists a solution for b, $b_m^T(B_m) \in (-\infty, \infty)$, to the equation $H(b, B_m) = \frac{C}{\alpha}$. Because $H(\cdot, \cdot)$ is monotonically increasing in b, that solution is unique. Thus, in this case,

$$\alpha E[\widetilde{B}|b, \widetilde{B} \geq B_m] \geq C \text{ if and only if } b \geq b_m^T(B_m),$$

so the executive's optimal approval function exists and is uniquely given by (17).

When $B_m > \frac{C}{\alpha}$, $H(b, B_m) > \frac{C}{\alpha}$, for all $b \in (-\infty, \infty)$, so $\alpha E[\widetilde{B}|b, \widetilde{B} \geq B_m] > C$ for all $b \in (-\infty, \infty)$, and the executive approves the proposed rule no matter the realization of b. In this case, then, define $b_m^T(B_m) = -\infty$, and again the executive has optimal approval behavior given by (17).

Finally, suppose $B_m = \frac{C}{\alpha}$. To show that the executive's behavior conforms to (17) in this case, we need to show that $\lim_{B_m \uparrow \frac{C}{\alpha}} b_m^T(B_m) = -\infty$, where $b_m^T(B_m)$ is the solution for b to the equation $H(b, B_m) = \frac{C}{\alpha}$ when $B_m < \frac{C}{\alpha}$. Consider, now, an arbitrary $M \in \mathbb{R}$, and evaluate $H(\cdot, \cdot)$ at $b = -M$ and $B_m = \frac{C}{\alpha}$. This gives us

$$H(-M, \frac{C}{\alpha}) = \frac{C}{\alpha} + \overline{\sigma}_m g\left(\frac{\frac{C}{\alpha} - \overline{B}_m(-M)}{\overline{\sigma}_m}\right) > \frac{C}{\alpha}, \tag{62}$$

where the inequality follows because $g(\cdot)$ is positive for any value of its argument. The inequality in (62) implies that there exists $\varepsilon > 0$ such that

$$H(-M, B_m) > \frac{C}{\alpha} \text{ for all } B_m \in \left(\frac{C}{\alpha} - \varepsilon, \frac{C}{\alpha}\right). \tag{63}$$

Because $H(\cdot, \cdot)$ is monotonically increasing in b, (63) implies that the solution for b to $H(b, B_m) = \frac{C}{\alpha}$ for all $B_m \in \left(\frac{C}{\alpha} - \varepsilon, \frac{C}{\alpha}\right)$ must be less than $-M$. That is, for any arbitrary $M \in \mathbb{R}$, there exists $\varepsilon > 0$ such that $b_m^T(B_m) < -M$ for all $B_m \in \left(\frac{C}{\alpha} - \varepsilon, \frac{C}{\alpha}\right)$. This is precisely the condition that $\lim_{B_m \uparrow \frac{C}{\alpha}} b_m^T(B_m) = -\infty$. ■

Proof of Proposition OA5. The equilibrium (B_m, ϕ_m) satisfies equations (12) and (21). Inverting $B_m(\phi_m)$ gives us $B_m^{-1}(B_m) = \frac{k}{\beta B_m - C}$, and the equilibrium condition can be written as a single equation in a single unknown, B_m:

$$B_m^{-1}(B_m) - \widehat{\Psi}\left(\frac{b_m^T(B_m) - B_m}{\sigma_b}\right) = 0.$$

Since the regulator will only propose a new rule if $B \geq \frac{C}{\beta} + \frac{k}{\beta\phi_m} \geq \frac{C+k}{\beta}$, the only pivotal benefits that are candidates for equilibrium are $B_m \in \left[\frac{C+k}{\beta}, \infty\right)$. Over this range, $B_m^{-1}(B_m)$ is continuous, strictly decreasing, with $B_m^{-1}(\frac{C+k}{\beta}) = 1$ and $\lim_{B_m\to\infty} B_m^{-1}(B_m) = 0$. Furthermore, as established in Lemma OA2, $b_m^T(\frac{C+k}{\beta}) \in (-\infty, \infty)$ exists and is unique for $B_m < \frac{C}{\alpha}$, $b_m^T(B_m) = -\infty$ for $B_m \geq \frac{C}{\alpha}$, and $\lim_{B_m \uparrow \frac{C}{\alpha}} b_m^T(B_m) = -\infty$. It is also straightforward to show that for $B_m < \frac{C}{\alpha}$, $b_m^T(B_m)$ is differentiable in B_m, with $\frac{db_m^T(B_m)}{dB_m}$ given by (19). This, in turn, implies that the function $\widehat{\Psi}\left(\frac{b_m^T(B_m) - B_m}{\sigma_b}\right)$ is continuous, $\lim_{B_m\to\infty} \widehat{\Psi}\left(\frac{b_m^T(B_m) - B_m}{\sigma_b}\right) = 0$ and $\widehat{\Psi}\left(\frac{b_m^T(B_m) - B_m}{\sigma_b}\right) = 1$ for $B_m \geq \frac{C}{\alpha}$. In addition, for $B_m < \frac{C}{\alpha}$, $\frac{d\widehat{\Psi}\left(\frac{b_m^T(B_m) - B_m}{\sigma_b}\right)}{dB_m} = \widehat{\Psi}'(\frac{b_m^T(B_m) - B_m}{\sigma_b})\left(\frac{db_m^T(B_m)}{dB_m} - 1\right) <$

0 since from (19), $\frac{db_m^T(B_m)}{dB_m} < 0$. Thus, $\widehat{\Psi}\left(\frac{b_m^T(B_m)-B_m}{\sigma_b}\right)$ is strictly increasing in B_m for $B_m \geq \frac{C}{\alpha}$.

Now, if $\alpha < \frac{C}{C+k}\beta$, the interval $\left(\frac{C+k}{\beta}, \frac{C}{\alpha}\right)$ is non-empty. Moreover,

$$\lim_{B_m \to \frac{C+k}{\beta}} B_m^{-1}(B_m) - \widehat{\Psi}\left(\frac{b_m^T(B_m) - B_m}{\sigma_b}\right) = 1 - \widehat{\Psi}\left(\frac{b_m^T(\frac{C+k}{\beta}) - \frac{C+k}{\beta}}{\sigma_b}\right) > 0.$$

$$\lim_{B_m \to \frac{C}{\alpha}} \left[B_m^{-1}(B_m) - \widehat{\Psi}\left(\frac{b_m^T(B_m) - B_m}{\sigma_b}\right)\right] = \frac{k}{\beta\frac{C}{\alpha} - C} - 1 < 0,$$

where the second inequality follows because $\alpha < \frac{C}{C+k}\beta \Leftrightarrow \frac{k}{\beta\frac{C}{\alpha}-C} < 1$. Since $B_m^{-1}(B_m)$ strictly decreases and $\widehat{\Psi}\left(\frac{b_m^T(B_m)-B_m}{\sigma_b}\right)$ strictly increases in B_m over the interval $\left(\frac{C+k}{\beta}, \frac{C}{\alpha}\right)$ and both functions are continuous, the intermediate value theorem and the monotonicity of $B_m^{-1}(B_m) - \widehat{\Psi}\left(\frac{b_m^T(B_m)-B_m}{\sigma_b}\right)$ implies there exists a unique $B_m^* \in \left(\frac{C+k}{\beta}, \frac{C}{\alpha}\right)$ such that $B_m^{-1}(B_m) = \widehat{\Psi}\left(\frac{b_m^T(B_m)-B_m}{\sigma_b}\right)$, or equivalently, we will have $B_m^* \in \left(\frac{C+k}{\beta}, \frac{C}{\alpha}\right)$ and $\phi_m^* \in (0,1)$ such that the equilibrium conditions (22) and (23) hold. It is then the case that $b_m^* = b_m^T(B_m^*) \in (-\infty, \infty)$. This establishes the first part of the proposition.

Suppose, now, $\alpha \geq \frac{C}{C+k}\beta$, or equivalently $\frac{C+k}{\beta} \geq \frac{C}{\alpha}$. As noted above the only pivotal benefits that are candidates for equilibrium are $B_m \in \left[\frac{C+k}{\beta}, \infty\right)$, but since $\frac{C+k}{\beta} \geq \frac{C}{\alpha}$, we have $\widehat{\Psi}\left(\frac{b_m^T(B_m)-B_m}{\sigma_b}\right) = 1$ for all $B_m \in \left[\frac{C+k}{\beta}, \infty\right)$. Recalling that $B_m^{-1}(B_m)$ is strictly decreasing with $B_m^{-1}(\frac{C+k}{\beta}) = 1$, there is a unique point of intersection between $\widehat{\Psi}\left(\frac{b_m^T(B_m)-B_m}{\sigma_b}\right)$ and $B_m^{-1}(B_m)$ at $B_m^* = \frac{C+k}{\beta}$ and $\phi_m^* = 1$. ■

Proof of Lemma OA3. Given Propositions OA4 and OA5, $\phi_n^* = \phi_m^* = 1$, $b_m^* = -\infty$, $B_m^* = \frac{C+k}{\beta} > \frac{C+k_r}{\beta} = B_n^* > \frac{C}{\alpha}$. It immediately follows from (27) that $\Delta_2 = 0$, and

$$\begin{aligned} EU_m^E - EU_n^E &= \Delta_1 + \Delta_3 + \Delta_4 \\ &= -\int_{\frac{C+k_r}{\beta}}^{\frac{C+k}{\beta}} [\alpha x - C - k_r] f_0(x)dx - k_a \int_{\frac{C+k}{\beta}}^{\infty} f_0(x)dx < 0 \end{aligned} \tag{64}$$

because the second term in (64) is clearly negative, and $\alpha \geq \beta$ ensures that over $x \in [\frac{C+k_r}{\beta}, \frac{C+k}{\beta}]$, $\alpha x - C - k_r > 0$, so the first term is negative as well. ■

Proof of Lemma OA4. When $\alpha > \frac{C}{C+k_r}\beta$, then given the implications of Propositions OA4 and OA5, $\frac{d\left[EU_m^E - EU_n^E\right]}{d\alpha} = -\int_{\frac{C+k_r}{\beta}}^{\frac{C+k}{\beta}} x f_0(x)dx < 0$, since $k_a > 0$ and thus $\frac{C+k_r}{\beta} < \frac{C+k}{\beta}$. ■

Proof of Proposition OA6. To prove (1), if $EU_m^E - EU_n^E < 0$ when $\alpha = \frac{C}{C+k_r}\beta$, then from Lemma OA4, it follows that $EU_m^E - EU_n^E < 0$ for all $\alpha \in (\frac{C}{C+k_r}\beta, \beta)$, and with Lemma OA3, that inequality then extends to $\alpha > \beta$.

To prove (2), if $EU_m^E - EU_n^E > 0$ when $\alpha = \frac{C}{C+k_r}\beta$, then since $EU_m^E - EU_n^E < 0$ at $\alpha = \beta$ and $EU_m^E - EU_n^E$ is continuous in α, the intermediate value theorem implies that there exists a point $\alpha^{**}(\beta) \in (\frac{C}{C+k_r}\beta, \beta)$ such that $EU_m^E - EU_n^E = 0$ at $\alpha = \alpha^{**}(\beta)$. Furthermore, by Lemma OA4, $EU_m^E - EU_n^E$ strictly decreases in α for $\alpha > \frac{C}{C+k_r}\beta$, so $\alpha^{**}(\beta)$ is unique and thus $EU_m^E - EU_n^E > 0$ for $\alpha \in \left(\frac{C}{C+k_r}\beta, \alpha^{**}(\beta)\right)$ and $EU_m^E - EU_n^E < 0$ for $\alpha > \alpha^{**}(\beta)$.

To derive the expression for $\alpha^{**}(\beta)$, we rearrange the expression for $EU_m^E - EU_n^E$ in (64) and solve for α, using expressions for truncated expectations for the standard normal distribution. ■

Proof of Lemma OA5. We refer to (31) and (33) as incentive compatibility (IC) conditions and (30) and (32) as individual rationality (IR) conditions. Given these conditions, for the regulator to propose with a

BCA for some values of B and without a BCA for other values of B, it must be the case that $\phi_v > 0$. If $\phi_v = 0$, then IR condition (30) would be violated. It must also be the case that $\phi_v < 1$. If $\phi_v = 1$ then because $E\left(\phi_v(\widetilde{b})|B\right) \leq 1$, and $k > k_r$, IC condition (33) cannot hold. Thus, only $\phi_v \in (0,1)$ is compatible with a separating equilibrium.

Furthermore, note that (33) implies that for any B such that a proposal is made with BCA, it must be the case that

$$\left[E\left(\phi_v(\widetilde{b})|B\right) - \phi_v\right](\beta B - C) \geq k - k_r = k_a > 0.$$

Because $\beta B > C$ for any proposal of any kind to be forthcoming, then it must be the case that $E\left(\phi_v(\widetilde{b})|B\right) > \phi_v$ for all B when a proposal with a BCA is submitted. Similarly, (31) implies that for B when a proposal is made without a BCA,

$$\left[E\left(\phi_v(\widetilde{b})|B\right) - \phi_v\right](\beta B - C) \leq k_a.$$

Because $\phi_v(b)$ is nondecreasing in b, it must be that $E\left(\phi_v(\widetilde{b})|B\right)$ is nondecreasing in B, since B shifts the distribution of $\widetilde{b}$ in the sense of first-order stochastic dominance. It then follows that both $\left[E\left(\phi_v(\widetilde{b})|B\right) - \phi_v\right](\beta B - C) - k_a$ and $E_{\widetilde{b}}\left(\phi_v(\widetilde{b})|B\right)(\beta B - C) - k$ are strictly increasing in B over the relevant range $B > \frac{C}{\beta}$ where the regulator would propose. Thus, there exists a unique value of B, call it B', at which $\left[E\left(\phi_v(\widetilde{b})|B'\right) - \phi_v\right](\beta B' - C) - k_a = 0$, and $\left[E\left(\phi_v(\widetilde{b})|B\right) - \phi_v\right](\beta B - C) - k_a \gtreqless 0$ as $B \gtreqless B'$. Analogously, there exists a unique value of B, denoted by B'', at which $E\left(\phi_v(\widetilde{b})|B\right)(\beta B - C) - k = 0$, and $E\left(\phi_v(\widetilde{b})|B\right)(\beta B - C) - k \gtreqless 0$ as $B \gtreqless B''$. Given this, we can express the set $\mathbf{B}_{NO}$ in which (30) and (31) hold as $\mathbf{B}_{NO} = \left\{B | B \geq \frac{C}{\beta} + \frac{k_r}{\beta\phi_v}, B \leq B'\right\}$, and we can express the set $\mathbf{B}_{BCA}$ in which (32) and (33) hold as $\mathbf{B}_{BCA} = \{B | B \geq B'', B \geq B'\} = \{B | B \geq \max\{B', B''\}\}$. Given this, if regulator's strategy satisfies the IC and IR conditions, and the executive receives a proposal from the regulator accompanied by a BCA with a measured benefit b, the executive's best response would be

$$\phi_v(b) = \begin{cases} 1 & \text{if } \alpha E[\widetilde{B}|b, \widetilde{B} \geq \max\{B', B''\}] > C \\ \in [0,1] & \text{if } \alpha E[\widetilde{B}|b, \widetilde{B} \geq \max\{B', B''\}] = C \\ 0 & \text{otherwise} \end{cases}.$$

We can use the same logic as in the proof of Lemma OA2 to show that $E[\widetilde{B}|b, \widetilde{B} \geq \max\{B', B''\}] - \frac{C}{\alpha}$ is strictly increasing in b. Thus, the executive's best-response approval rule $\phi_v(b)$ is non-decreasing in b, consistent with the regulator's expectations, and it involves a unique threshold benefit b_v (possibly equal to $-\infty$).

Now consider the executive's optimal approval behavior in a separating equilibrium if it receives a proposal *without* a BCA. Because we have shown that $\phi_v \in (0,1)$, it must be that if the executive receives a proposal without a BCA

$$E[\widetilde{B}|\widetilde{B} \in \mathbf{B}_{NO}] = \frac{C}{\alpha} \leq B', \tag{65}$$

where the inequality follows because for all $B \in \mathbf{B}_{NO}$, $B \leq B'$, so $E[\widetilde{B}|\widetilde{B} \in \mathbf{B}_{NO}] \leq B'$.

Now because $B \in \mathbf{B}_{BCA} \Rightarrow B \geq B'$, it follows that $E[\widetilde{B}|b, \widetilde{B} \in \mathbf{B}_{BCA}] > B'$. But given (65), we then have

$$E[\widetilde{B}|b, \widetilde{B} \in \mathbf{B}_{BCA}] > \frac{C}{\alpha}.$$

This implies that the executive would always approve a proposal with a BCA for any measured benefit b. Thus, $\phi_v(b) = 1$ for all $b \in (-\infty, \infty)$, and the measured benefit threshold $b_v = -\infty$.

Given $\phi_v(b) = 1$ for any b, it is easy to see that $B' = \frac{C}{\beta} + \frac{k_a}{\beta(1-\phi_v)}$ and $B'' = \frac{C+k}{\beta}$, and thus

$$\mathbf{B}_{NO} = \left\{ B | B \in \left[\frac{C}{\beta} + \frac{k_r}{\beta \phi_v}, \frac{C}{\beta} + \frac{k_a}{\beta(1-\phi_v)} \right] \right\}.$$
$$\mathbf{B}_{BCA} = \left\{ B | B \geq \frac{C+k}{\beta}, B \geq \frac{C}{\beta} + \frac{k_a}{\beta(1-\phi_v)} \right\}.$$

For a separating equilibrium to exist the set $\mathbf{B}_{NO}$ must be non-empty, i.e., $\frac{C}{\beta} + \frac{k_r}{\beta\phi_v} < \frac{C}{\beta} + \frac{k_a}{\beta(1-\phi_v)}$, or

$$\phi_v > \frac{k_r}{k}. \tag{66}$$

It follows directly that $\frac{C}{\beta} + \frac{k_a}{\beta(1-\phi_v)} - \frac{C+k}{\beta} > 0$, so $\mathbf{B}_{BCA} = \left\{ B | B \in \left[\frac{C}{\beta} + \frac{k_a}{\beta(1-\phi_v)}, \infty \right] \right\}$. ■

Proof of Proposition OA7. Let $B_v^+(\phi) = \frac{C}{\beta} + \frac{k_a}{\beta(1-\phi)}$, $B_v^-(\phi) = \frac{C}{\beta} + \frac{k_r}{\beta\phi}$, and let $\overline{B}(\phi) \equiv E[\widetilde{B}|\widetilde{B} \in [B_v^-(\phi), B_v^+(\phi)]]$. We can write the equilibrium condition (36) as

$$\alpha \overline{B}(\phi) = C. \tag{67}$$

A separating equilibrium exists if (67) holds for some $\phi \in \left(\frac{k_r}{k}, 1\right)$.

As a first step, note that $\overline{B}(1) = E[\widetilde{B}|\widetilde{B} \in [\frac{C+k_r}{\beta}, \infty)] = B_0 + \sigma_0 h \left(\frac{\frac{C+k_r}{\beta} - B_0}{\sigma_0} \right)$, using the formula for the one-sided truncation of the normal distribution. Note, too, that $B_v^+(\frac{k_r}{k}) = \frac{C}{\beta} + \frac{k_a}{\beta(1-\frac{k_r}{k})} = B_v^-(\frac{k_r}{k}) = \frac{C}{\beta} + \frac{k_r}{\beta\frac{k_r}{k}} = \frac{C+k}{\beta}$. Thus, $\overline{B}(\frac{k_r}{k}) = E[\widetilde{B}|\widetilde{B} \in [\frac{C+k}{\beta}, \frac{C+k}{\beta}]] = \frac{C+k}{\beta}$. Recalling that $\alpha_n^*(\beta) \equiv \frac{C}{B_0 + \sigma_0 h\left(\frac{\frac{C+k_r}{\beta} - B_0}{\sigma_0}\right)}$ and $\alpha_m^*(\beta) = \frac{C}{C+k}\beta$, we have

$$\overline{B}(1) = \frac{C}{\alpha_n^*(\beta)}. \tag{68}$$
$$\overline{B}\left(\frac{k_r}{k}\right) = \frac{C}{\alpha_m^*(\beta)}. \tag{69}$$

Suppose that $\alpha_n^*(\beta) \neq \alpha_m^*(\beta)$. This implies

$$\min\{\alpha_n^*(\beta), \alpha_m^*(\beta)\} < \max\{\alpha_n^*(\beta), \alpha_m^*(\beta)\},$$

and thus

$$\min\left\{\overline{B}(1), \overline{B}\left(\frac{k_r}{k}\right)\right\} < \max\left\{\overline{B}(1), \overline{B}\left(\frac{k_r}{k}\right)\right\}.$$

This, in turn, implies the result stated in the proposition, $\overline{B}^{\min} < \overline{B}^{\max}$ since:

$$\overline{B}^{\min} = \min_{\phi \in \left[\frac{k_r}{k}, 1\right]} \overline{B}(\phi) \leq \min\left\{\overline{B}(1), \overline{B}\left(\frac{k_r}{k}\right)\right\}$$
$$< \max\left\{\overline{B}(1), \overline{B}\left(\frac{k_r}{k}\right)\right\} \leq \max_{\phi \in \left[\frac{k_r}{k}, 1\right]} \overline{B}(\phi) = \overline{B}^{\max}.$$

Next, consider any $\alpha \in \left(\frac{C}{\overline{B}^{\max}}, \frac{C}{\overline{B}^{\min}}\right)$, or equivalently, any $\frac{C}{\alpha} \in \left(\overline{B}^{\min}, \overline{B}^{\max}\right)$. Because $\overline{B}(\phi)$ is continuous on the interval $\left(\frac{k_r}{k}, 1\right)$, the intermediate value theorem implies that there exists a value ϕ_v such that $\overline{B}(\phi_v) = \frac{C}{\alpha}$. That is, a solution to (67) exists on $\left(\frac{k_r}{k}, 1\right)$.

Suppose, now, $\alpha > \frac{C}{\overline{B}^{\min}}$. Then $\alpha\overline{B}(\phi) - C > 0$ for all $\phi \in \left[\frac{k_r}{k}, 1\right]$,and the equilibrium condition (67) for a separating equilibrium cannot hold on $\left(\frac{k_r}{k},1\right)$. Likewise, if $\alpha < \frac{C}{\overline{B}^{\max}}$, then $\alpha\overline{B}(\phi) - C < 0$ for all $\phi \in \left[\frac{k_r}{k},1\right]$, and (67) cannot hold on $\left(\frac{k_r}{k},1\right)$. Thus, if $\alpha > \frac{C}{\overline{B}^{\min}}$ or if $\alpha < \frac{C}{\overline{B}^{\max}}$, a separating equilibrium in the regulatory proposal subgame with voluntary BCA cannot exist.

To complete the proof, suppose $\alpha = \frac{C}{\overline{B}^{\min}}$ or $\alpha = \frac{C}{\overline{B}^{\max}}$. We will structure the argument for the case of $\alpha = \frac{C}{\overline{B}^{\min}}$, noting that the logic for $\alpha = \frac{C}{\overline{B}^{\max}}$ is identical. Let $\phi^{\min}$ be a value of ϕ that attains $\overline{B}^{\min}$. If $\phi^{\min} \in (\frac{k_r}{k},1)$, then a separating equilibrium exists at $\alpha = \frac{C}{\overline{B}^{\min}}$. If either $\phi^{\min} = 1$ or $\phi^{\min} = \frac{k_r}{k}$, and a separating equilibrium would not exist for $\alpha = \frac{C}{\overline{B}^{\min}}$. In this case of non-existence, we would have to have $\overline{B}^{\min} = \overline{B}(1)$ or $\overline{B}^{\min} = \overline{B}\left(\frac{k_r}{k}\right)$. But then $\alpha = \frac{C}{\overline{B}^{\min}} \implies \alpha = \frac{C}{\overline{B}(1)}$ or $\alpha = \frac{C}{\overline{B}(\frac{k_r}{k})}$, and from (68) and (69), it would follow that $\alpha = \alpha_n^*(\beta)$ or $\alpha = \alpha_m^*(\beta)$. Thus, if $\frac{C}{\overline{B}^{\min}} = \alpha_n^*(\beta)$ or $\frac{C}{\overline{B}^{\min}} = \alpha_m^*(\beta)$ a separating equilibrium does not exist for $\alpha = \frac{C}{\overline{B}^{\min}}$. By analogous logic, if $\frac{C}{\overline{B}^{\max}} = \alpha_n^*(\beta)$ or $\frac{C}{\overline{B}^{\max}} = \alpha_m^*(\beta)$ a separating equilibrium does not exist for $\alpha = \frac{C}{\overline{B}^{\max}}$. Summing up this logic, a separating equilibrium exists for $\alpha = \frac{C}{\overline{B}^{\min}}$ unless$\frac{C}{\overline{B}^{\min}} = \alpha_n^*(\beta)$ or $\frac{C}{\overline{B}^{\min}} = \alpha_m^*(\beta)$, and a separating equilibrium exists for $\alpha = \frac{C}{\overline{B}^{\max}}$ unless$\frac{C}{\overline{B}^{\max}} = \alpha_n^*(\beta)$ or $\frac{C}{\overline{B}^{\max}} = \alpha_m^*(\beta)$. ■

Proof of Proposition OA8. The biased BCA can be "de-biased" by subtracting A from the realized value of $\widetilde{b}_a$. Because $\widetilde{b}_a - A$ is normally distributed with mean B and standard deviation σ_b, the posterior distribution of $\widetilde{B}$ conditional on a realization $b_a - A$ of the de-biased BCA is normal with variance $\overline{\sigma}_a^2 = \frac{\sigma_b^2\sigma_0^2}{\sigma_b^2+\sigma_0^2}$ and mean

$$\overline{B}_a(b,A) = \frac{\sigma_b^2}{\sigma_b^2+\sigma_0^2}B_0 + \frac{\sigma_0^2}{\sigma_b^2+\sigma_0^2}\left(b_A - A\right).$$

By totally differentiating the expression for $b_a^T(B_a, A)$ with respect to A, it is straightforward to show that for any arbitrary pivotal benefit B_a, $b_a^T(B_a, A) = b_m^T(B_a) + A$. The equilibrium is then the triple (B_a, ϕ_a, b_a) that solves then solves

$$B_a = \frac{C}{\beta} + \frac{k}{\beta\phi_a},$$
$$\phi_a = \widehat{\Psi}\left(\frac{b_a - B_a - A}{\sigma_b}\right),$$
$$b_a = b_m^T(B_a + A.$$

The solution is clearly $B_a^* = B_m^*$, $\phi_a^* = \phi_m^*$, $b_a^* = b_m^* + A$. ■

Proof of Lemma OA6.

Sufficiency: Because $\alpha \geq \beta$ and $\frac{dB_{sa}(A)}{dA} < 0$, $MC(A,\alpha) \leq 0$ for all $A \in (-\infty,\infty)$. To evaluate $MB(A,\alpha)$, we know $B_{sa}(A) \geq \frac{C+k}{\beta}$, so if $\alpha \geq \beta$,

$$B_{sa}(A) \geq \frac{C+k}{\beta} \geq \frac{C+k}{\alpha} > \frac{C}{\alpha},$$

and thus the integral in (40) is positive. It follows that $MB(A,\alpha) - MC(A,\alpha) > 0$ for all A when $\alpha \geq \beta$, and thus the unique optimum for the executive's problem is $A^* = \infty$ and $B_{sa}^* = \frac{C+k}{\beta}$.

Necessity: For any arbitrary value of A, let $\widehat{\alpha}(A)$ be a solution for α to $MB(A,\alpha) = MC(A,\alpha)$. This solution exists and is unique for the following reasons: (i) We can see directly from (40) and (41) that $MB(A,\alpha)- MC(A,\alpha)$ is strictly increasing in α. (ii) We just established $MB(A,\alpha) - MC(A,\alpha) > 0$ for all A when $\alpha \geq \beta$. (iii) We can see from (40) and (41) that $MB(A,0) < 0 \leq MC(A,0)$ for any A, so $MB(A,0) - MC(A,0) < 0$. (iv) Given (i), (ii), and (iii), the intermediate value theorem implies that for any A, there exists a unique solution for α to $MB(A,\alpha) = MC(A,\alpha)$.

To establish necessity, it suffices to show that $\lim_{A\to\infty}\widehat{\alpha}(A) = \beta$. Suppose, to the contrary, this is not the case. Then $\lim_{A\to\infty}\ MB(A,\alpha)|_{\alpha=\beta} \neq \lim_{A\to\infty}\ MC(A,\alpha)|_{\alpha=\beta}$. From (41), $MC(A,\alpha)|_{\alpha=\beta} = 0$.

To obtain a contradiction and establish the result, it suffices to show that $\lim_{A\to\infty} MB(A,\alpha) = 0$ for any α (thus including $\alpha = \beta$). From the definition of $B_{sa}(A)$, we can see that $\lim_{A\to\infty} B_{sa}(A) = \frac{C+k}{\beta}$. Furthermore, since $\psi(\cdot)$ is the standard normal density function, $\lim_{A\to\infty} \psi\left(\frac{C-A-x}{\sigma_b}\right) = 0$. Hence, from (40), $\lim_{A\to\infty} MB(A,\alpha) = 0$ for any α. This contradicts the implication of the contrapositive assumption, $\lim_{A\to\infty} \widehat{\alpha}(A) \neq \beta$, thus establishing that when $\alpha = \beta$ the optimal solution to the executive's problem is $A^* = \infty$. Given that A^* increases with α, this implies that $A^*(\alpha) = \infty$ for all $\alpha > \beta$ as well. ■

Proof of Proposition OA10:

Preliminary steps: We begin by noting that $\overline{x}_m(\beta) = \frac{k}{\beta B^L - C} - (1-q) > -(1-q)$. Moreover, if $\beta < \frac{k}{2(1-q)B^L}$, then $\overline{x}_m(\beta) > 1-q$. Let us now define two additional objects: $\overline{x}_m(\alpha, b^L)$ is the inverse of $\overline{\alpha}_m(b^L, x)$ with respect to x, and is $\overline{x}_m(\alpha, b^H)$ the inverse of $\overline{\alpha}_m(b^H, x)$ with respect to x. We have

$$\overline{x}_m(\alpha, b^L) = \frac{(1-q)p(\alpha B^H - C) - q(1-p)(C - \alpha B^L)}{p(\alpha B^H - C) - (1-p)(C - \alpha B^L)} = \frac{\alpha E[\widetilde{B}|b^L] - C}{\alpha E[\widetilde{B}] - C}.$$

$$\overline{x}_m(\alpha, b^H) = -\frac{qp(\alpha B^H - C) - (1-q)(1-p)(C - \alpha B^L)}{p(\alpha B^H - C) - (1-p)(C - \alpha B^L)} = -\frac{\alpha E[\widetilde{B}|b^H] - C}{\alpha E[\widetilde{B}] - C}.$$

Because $E[\widetilde{B}|\widetilde{b} = b^L, x]$ and $E[\widetilde{B}|\widetilde{b} = b^H, x]$ are strictly decreasing in x, and $\overline{\alpha}_m(b^L, x) = \frac{C}{E[\widetilde{B}|\widetilde{b}=b^L,x]}$ and $\overline{\alpha}_m(b^H, x) = \frac{C}{E[\widetilde{B}|\widetilde{b}=b^H,x]}$ are thus strictly increasing in x, it follows that $\overline{x}_m(\alpha, b^L)$ and $\overline{x}_m(\alpha, b^H)$ are strictly increasing in α. In fact, $\overline{x}_m(\alpha, b^L)$ and $\overline{x}_m(\alpha, b^H)$ are hyperbolas in (α, x) space, with each having an asymptote toward ∞ and $-\infty$ at $\alpha = \overline{\alpha}_n$. Straightforward algebra implies that $\overline{x}_m(\alpha, b^L) - \overline{x}_m(\alpha, b^H) = 1$ for all $\alpha \in \left(\frac{C}{B^H}, \frac{C}{B^L}\right)$ except at $\alpha = \overline{\alpha}_n$ where the difference is not defined. Moreover,

$$\overline{x}_m(\alpha, b^L) > 1-q,\ \alpha \in \left(\frac{C}{B^H}, \overline{\alpha}_n\right). \tag{70}$$

$$\overline{x}_m(\alpha, b^L) \in (-\infty, 1-q)],\ \alpha \in \left(\overline{\alpha}_n, \frac{C}{B^L}\right). \tag{71}$$

Analogously,

$$\overline{x}_m(\alpha, b^H) \in [-(1-q), \infty),\ \ \alpha \in \left(\frac{C}{B^H}, \overline{\alpha}_n\right). \tag{72}$$

$$\overline{x}_m(\alpha, b^H) < -(1-q),\ \alpha \in \left(\overline{\alpha}_n, \frac{C}{B^L}\right). \tag{73}$$

Now, consider the expressions in (44) and in particular the difference between the third expression in (44) and the last and next-to-last expressions, respectively. We denote these differences $\Gamma_1(x)$ and $\Gamma_2(x)$. We will show that for any x and α such that $\alpha \leq \overline{\alpha}_m(b^L, x)$

$$\Gamma_1(x) = p\left(\frac{2q-1}{q-x}\right)(\alpha B^H - C) - pk\left(1 + \frac{(1-q-x)(\alpha B^H - C)}{(q-x)(C - \alpha B^L)}\right) - \left[-kp\left(1 + \frac{(q+x)(\alpha B^H - C)}{(1-q+x)(C-\alpha B^L)}\right)\right] > 0,$$

and

$$\Gamma_2(x) = p\left(\frac{2q-1}{q-x}\right)(\alpha B^H - C) - pk\left(1 + \frac{(1-q-x)(\alpha B^H - C)}{(q-x)(C - \alpha B^L)}\right) - \left\{\left[\begin{array}{c} p(q+x)(\alpha B^H - C) \\ -(1-p)(1-q+x)(C - \alpha B^L)\end{array}\right] - k\right\} > 0. \tag{74}$$

For $\Gamma_1(x)$ we have

$$\Gamma_1(x) > kp\left[\frac{\alpha B^H - C}{C - \alpha B^L}\right]\left(\frac{(q+x)}{(1-q+x)} - \frac{(1-q-x)}{(q-x)}\right) > 0,$$

where the first inequality follows because (i) $q > \frac{1}{2}$, (ii) $\alpha B^H - C > 0$ (iii) $q - x >$ since $x \leq 1 - q < q$. The second inequality follows because $\frac{q+x}{(1-q+x)} - \frac{(1-q-x)}{(q-x)} = \frac{q^2-(1-q^2)}{(1-q+x)(q-x)} = \frac{2q-1}{(1-q+x)(q-x)} > 0$. For $\Gamma_2(x)$ note that

$$\begin{aligned} & k - p\left(1 + \frac{(1-q-x)(\alpha B^H - C)}{(q-x)\,(C - \alpha B^L)}\right)k \\ &= k(1-p)\left\{1 - \frac{p}{1-p}\frac{(1-q-x)(\alpha B^H - C)}{(q-x)\,(C-\alpha B^L)}\right\}. \end{aligned}$$

Because $\overline{\alpha}_m(b^L, x) = \frac{C}{E[\widetilde{B}|\widetilde{b}=b^L,x]}$, using the expression (43) for $E[\widetilde{B}|\widetilde{b} = b^L, x]$, we have

$$\frac{p}{1-p}\frac{(1-q-x)(\overline{\alpha}_m(b^L,x)B^H - C)}{(q-x)\,(C - \overline{\alpha}_m(b^L,x)B^L)} = 1,$$

and thus $1 - \frac{p}{1-p}\frac{(1-q-x)(\alpha B^H - C)}{(q-x)(C-\alpha B^L)} > 0$ for any x and α such that $\alpha < \overline{\alpha}_m(b^L, x)$ because $\frac{\alpha B^H - C}{C - \alpha B^L}$ is an increasing function of α. Thus, the term

$$-pk\left(1 + \frac{(1-q-x)\,(\alpha B^H - C)}{(q-x)\,(C-\alpha B^L)}\right) - (-k)$$

in (74) is positive, so

$$\Gamma_2(x) > p\left(\frac{2q-1}{q-x}\right)(\alpha B^H - C) - \left[p(q+x)(\alpha B^H - C) - (1-p)(1-q+x)(C - \alpha B^L)\right].$$

Straightforward algebra enables us to rewrite the right-hand side of this expression as

$$\frac{(1-q+x)}{q-x)}(C - \alpha B^L)\left\{1 - \frac{(\alpha B^H - C)}{(C-\alpha B^L)}\frac{p}{1-p}\frac{(1-q-x)}{(q-x)} + 1\right\} > 0,$$

where the inequality follows because $1 - \frac{p}{1-p}\frac{(1-q-x)(\alpha B^H - C)}{(q-x)(C-\alpha B^L)} > 0$ for $\alpha < \overline{\alpha}_m(b^L, x)$, or equivalently, $x < \overline{x}_m(\alpha, b^L)$. Thus, $\Gamma_2(x) > 0$.

Next, consider how the last three expressions in (44) vary with x:

$$\begin{aligned} & \frac{d\left[p\left(\frac{2q-1}{q-x}\right)(\alpha B^H - C) - pk\left(1 + \frac{(1-q-x)(\alpha B^H - C)}{(q-x)(C-\alpha B^L)}\right)k\right]}{dx} \\ &= \frac{p(2q-1)(\alpha B^H - C)}{(q-x)^2} + \frac{pk(2q-1)}{(q-x)^2}\left(\frac{\alpha B^H - C}{C - \alpha B^L}\right) > 0. \end{aligned}$$

$$\frac{d\left[-kp\left(1 + \frac{(q+x)(\alpha B^H - C)}{(1-q+x)(C-\alpha B^L)}\right)\right]}{dx} = pk\frac{(\alpha B^H - C)}{(C - \alpha B^L)}\frac{(2q-1)}{(1-q+x)^2} > 0. \tag{75}$$

and

$$\begin{aligned} & \frac{d\left[p(q+x)(\alpha B^H - C) + (1-p)(1-q+x)(\alpha B^L - C) - k\right]}{dx} \\ &= p(\alpha B^H - C) - (1-p)(C - \alpha B^L) \gtreqless 0 \text{ as } \alpha \gtreqless \overline{\alpha}_n \end{aligned} \tag{76}$$

Note, too that for α and x such that $\alpha = \overline{\alpha}_m(b^H, x)$, then $\alpha = \frac{C}{E[\widetilde{B}|\widetilde{b}=b^H,x]}$ and from (42), $p(q+x)(\alpha B^H - C) =$

$(1-p)(1-q+x)(\alpha B^L - C)$, and thus

$$p(q+x)(\alpha B^H - C) + (1-p)(1-q+x)(\alpha B^L - C) - k = -k. \tag{77}$$

Let's also observe that for α and x such that $\alpha < \overline{\alpha}_m(b^H, x)$, from (44), $EU_a^E(x) < 0$. Given the continuity of $EU_a^E(x)$ in x for $x > \overline{x}_m(\beta)$ (equivalently $\beta > \overline{\beta}_m(x)$), along with (76), (77), it follows that for $x > \overline{x}_m(\beta)$ and $\alpha \in \left(\frac{C}{B^H}, \overline{\alpha}_n\right]$, that $EU_a^E(x) < 0$.

Proof of (a): If $\beta \in \left(\frac{C+k}{B^L}, \frac{C}{B^L} + \frac{k}{2(1-q)B^L}\right)$, then as noted earlier in the proof, $\overline{x}_m(\beta) > 1-q$, which means that for $x \in [-(1-q), 1-q]$

$$EU_a^E(x) = \begin{cases} \alpha(pB^H + (1-p)B^L) - C - k & \alpha \geq \overline{\alpha}_m(b^L, x) \\ p\left(\left(\frac{2q-1}{q-x}\right)(\alpha B^H - C) - \left(1 + \frac{(1-q-x)(\alpha B^H - C)}{(q-x)(C-\alpha B^L)}\right)k\right) & \alpha \leq \overline{\alpha}_m(b^L, x) \end{cases}.$$

Now, for $\alpha \in \left(\frac{C}{B^H}, \overline{\alpha}_n\right]$, it is not possible for $\alpha \geq \overline{\alpha}_m(b^L, x)$ since $\overline{\alpha}_n < \overline{\alpha}_m(b^L, x)$ for all $x \in [-(1-q), 1-q]$. Thus, $EU_a^E(x) = p\left(\left(\frac{2q-1}{q-x}\right)(\alpha B^H - C) - \left(1 + \frac{(1-q-x)(\alpha B^H - C)}{(q-x)(C-\alpha B^L)}\right)k\right)$, which, as we have seen in "preliminary steps," is a strictly increasing function, which is maximized at $x = 1-q$. If $\alpha \in \left(\overline{\alpha}_n, \frac{C}{B^L}\right)$, then from (71), $\overline{x}_m(\alpha, b^L) \in (-\infty, 1-q)]$, and thus $EU_a^E(x)$ can be written

$$EU_a^E(x) = \begin{cases} \alpha(pB^H + (1-p)B^L) - C - k & x \leq \overline{x}_m(\alpha, b^L) \\ p\left(\left(\frac{2q-1}{q-x}\right)(\alpha B^H - C) - \left(1 + \frac{(1-q-x)(\alpha B^H - C)}{(q-x)(C-\alpha B^L)}\right)k\right) & x \geq \overline{x}_m(\alpha, b^L) \end{cases}.$$

We can see that this function, too, is maximized at $x = 1-q$.

Proof of (b): If $\beta > \frac{C}{B^L} + \frac{k}{2(1-q)B^L}$, then $-(1-q) < \overline{x}_m(\beta) < 1-q$. Because $\alpha \in \left(\frac{C}{B^H}, \overline{\alpha}_n\right)$, then it is not possible for $\alpha \geq \overline{\alpha}_m(b^L, x)$ since $\overline{\alpha}_n < \overline{\alpha}_m(b^L, x)$. Moreover, from (71) and (72), $\overline{x}_m(\alpha, b^L) > 1-q$ and $\overline{x}_m(\alpha, b^H) \in [-(1-q), \infty)$. Thus, we can write $EU_a^E(x)$ as follows:

$$EU_a^E(x) = \begin{cases} p\left(\left(\frac{2q-1}{q-x}\right)(\alpha B^H - C) - \left(1 + \frac{(1-q-x)(\alpha B^H - C)}{(q-x)(C-\alpha B^L)}\right)k\right) & x \leq \overline{x}_m(\beta) \\ p(q+x)(\alpha B^H - C) + (1-p)(1-q+x)(\alpha B^L - C) - k & x > \overline{x}_m(\beta),\ x \leq \overline{x}_m(b^H, x) \\ -kp\left(1 + \frac{(q+x)(\alpha B^H - C)}{(1-q+x)(C-\alpha B^L)}\right) & x > \overline{x}_m(\beta), x \geq \overline{x}_m(b^H, x) \end{cases}. \tag{78}$$

This gives us three cases: (i) $-(1-q) < \overline{x}_m(\alpha, b^H) < \overline{x}_m(\beta) < 1-q$; (ii) $-(1-q) < \overline{x}_m(\beta) < \overline{x}_m(\alpha, b^H) < 1-q$; (iii) $-(1-q) < \overline{x}_m(\beta) < 1-q < \overline{x}_m(\alpha, b^H)$. Consider each in turn.

Case (i): $-(1-q) < \overline{x}_m(\alpha, b^H) < \overline{x}_m(\beta) < 1-q$. In this case $EU_a^E(x)$ jumps downward at $\overline{x}_m(\beta)$ to $p(q+x)(\alpha B^H - C) + (1-p)(1-q+x)(\alpha B^L - C) - k$, which as noted in (76) is strictly decreasing in x when $\alpha \in \left(\frac{C}{B^H}, \overline{\alpha}_n\right)$. In this circumstance, $\overline{x}_m(\beta)$ is the unique maximizer of $EU_a^E(x)$ on $[-(1-q), 1-q]$.

Case (ii): $-(1-q) < \overline{x}_m(\beta) < \overline{x}_m(\alpha, b^H) < 1-q$. In this case, $EU_a^E(x)$ jumps downward to $EU_a^{E-}(x) \equiv -kp\left(1 + \frac{(q+x)(\alpha B^H - C)}{(1-q+x)(C-\alpha B^L)}\right)$, which from (75) is strictly increasing at in x, but once x increases beyond $\overline{x}_m(\alpha, b^H)$, $EU_a^E(x)$ becomes $(q+x)(\alpha B^H - C) + (1-p)(1-q+x)(\alpha B^L - C) - k$, which is strictly decreasing in x. Thus, $EU_a^{E-}(x)$ attains its maximum at $\overline{x}_m(\alpha, b^H)$. In this case, the unique maximizer of $EU_a^E(x)$ is either $\overline{x}_m(\beta)$ or $\overline{x}_m(\alpha, b^H)$ depending on whether $EU_a^E(\overline{x}_m(\beta))$ is greater than or less than $EU_a^E(\overline{x}_m(\alpha, b^H))$. But as noted in (77), $p(q+x)(\alpha B^H - C) + (1-p)(1-q+x)(\alpha B^L - C) - k = -k$ at $\alpha = \overline{\alpha}_m(b^H, x)$, or equivalently, $x = \overline{x}_m(\alpha, b^H)$. Thus, $EU_a^{E-}(\overline{x}_m(\alpha, b^H)) = -k$, and so

$$\begin{aligned} EU_a^E(\overline{x}_m(\beta)) - EU_a^{E-}(\overline{x}_m(\alpha, b^H)) &= p\left\{ \begin{array}{c} \left(\frac{2q-1}{q-\overline{x}_m(\beta)}\right)(\alpha B^H - C) \\ -\left(1 + \frac{(1-q-\overline{x}_m(\beta))(\alpha B^H - C)}{(q-\overline{x}_m(\beta))(C-\alpha B^L)}\right)k \end{array} \right\} + k \\ &> k - pk\left(1 + \frac{(1-q-\overline{x}_m(\beta))(\alpha B^H - C)}{(q-\overline{x}_m(\beta))(C-\alpha B^L)}\right) > 0, \end{aligned}$$

where the first inequality follows because $q > \frac{1}{2}$, and the second inequality follows because, as part of the proof above that $\Gamma_2(x) > 0$, we showed that $k - pk\left(1 + \frac{(1-q-x))(\alpha B^H - C)}{(q-x)(C-\alpha B^L)}\right) > 0$ for any $x < \overline{x}_m(\alpha, b^L)$, and when $\alpha \in \left(\frac{C}{B^H}, \overline{\alpha}_n\right)$, $\overline{x}_m(\beta) < \overline{x}_m(\alpha, b^L)$. Thus, when $\alpha \in \left(\frac{C}{B^H}, \overline{\alpha}_n\right)$ and $\overline{x}_m(\beta) < \overline{x}_m(\alpha, b^H) < 1-q$, the unique optimal solution to the executive's problem is $\overline{x}_m(\beta)$.

Case (iii): $-(1-q) < \overline{x}_m(\beta) < 1-q < \overline{x}_m(\alpha, b^H)$. In this case, $EU_a^E(x)$ jumps downward at $\overline{x}_m(\beta)$ to $EU_a^E(x) = -kp\left(1 + \frac{(q+x)(\alpha B^H - C)}{(1-q+x)(C-\alpha B^L)}\right)$, which is strictly increasing in x. In this case, the unique maximizer of $EU_a^E(x)$ is either $\overline{x}_m(\beta)$ or $1-q$ depending on whether $EU_a^E(\overline{x}_m(\beta))$ is greater than or less than $-kp\left(1 + \frac{(q+x)(\alpha B^H - C)}{(1-q+x)(C-\alpha B^L)}\right)\Big|_{x=1-q}$. Now $-kp\left(1 + \frac{(q+x)(\alpha B^H - C)}{(1-q+x)(C-\alpha B^L)}\right)\Big|_{x=\overline{x}_m(\alpha,b^H)} = -kp\left(1 + \frac{(q+x)(\overline{\alpha}_m(b^H,x)B^H - C)}{(1-q+(\overline{\alpha}_m(b^H,x)(C-\alpha B^L)}\right)$. Recalling that $\overline{\alpha}_m(b^H, x) = \frac{C}{E[B|b^H,x]}$, (42) implies

$$\frac{(q+x)(\overline{\alpha}_m(b^H,x)B^H - C)}{(1-q+(\overline{\alpha}_m(b^H,x)(C-\alpha B^L)} = \frac{1-p}{p},$$

and thus

$$-kp\left(1 + \frac{(q+x)(\alpha B^H - C)}{(1-q+x)(C-\alpha B^L)}\right)\Big|_{x=\overline{x}_m(\alpha,b^H)} = -k.$$

Because $-kp\left(1 + \frac{(q+x)(\alpha B^H - C)}{(1-q+x)(C-\alpha B^L)}\right)$ increases in x and we have supposed $1-q < \overline{x}_m(\alpha, b^H)$, then $-kp\left(1 + \frac{(q+x)(\alpha B^H - C)}{(1-q+x)(C-\alpha B^L)}\right)\Big|_x$ $-k$. Thus

$$\begin{aligned}
&EU_a^E(\overline{x}_m(\beta)) - -kp\left(1 + \frac{(q+x)(\alpha B^H - C)}{(1-q+x)(C-\alpha B^L)}\right)\Big|_{x=1-q} \\
&> EU_a^E(\overline{x}_m(\beta)) + k, \\
&= p\left\{\left(\frac{2q-1}{q-\overline{x}_m(\beta)}\right)(\alpha B^H - C) - \left(1 + \frac{(1-q-\overline{x}_m(\beta))(\alpha B^H - C)}{(q-\overline{x}_m(\beta))(C-\alpha B^L)}\right)k\right\} + k \\
&> k - pk\left(1 + \frac{(1-q-\overline{x}_m(\beta))(\alpha B^H - C)}{(q-\overline{x}_m(\beta))(C-\alpha B^L)}\right) \\
&> 0,
\end{aligned}$$

where the second and third inequalities follow from the same arguments we made to show that $EU_a^{E+}(x) - EU_a^{E-}(\overline{x}_m(\alpha, b^H)) > 0$ in case (ii). Thus, when $\alpha \in \left(\frac{C}{B^H}, \overline{\alpha}_n\right)$, $\overline{x}_m(\beta) < 1-q < \overline{x}_m(\alpha, b^H)$, $EU_a^E(\overline{x}_m(\beta)) - EU_a^E(1-q) > 0$, so $\overline{x}_m(\beta)$ is the unique maximizer of $EU_a^E(x)$ on $[-(1-q), 1-q]$.

Summing up, we have thus established in all possible cases that can arise when $\alpha \in \left(\frac{C}{B^H}, \overline{\alpha}_n\right)$, $\overline{x}_m(\beta)$ is the unique maximizer of $EU_a^E(x)$ on $[-(1-q), 1-q]$.

Proof of (c): If $\alpha = \overline{\alpha}_n$, it is necessarily the case that $\overline{\alpha}_m(b^H, x) < \alpha < \overline{\alpha}_m(b^L, x)$ for any x. Thus, at $\overline{x}_m(\beta)$, $EU_a^E(x)$ jumps downward to $EU_a^E(x) = p(q+x)(\alpha B^H - C) + (1-p)(1-q+x)(\alpha B^L - C) - k$. This is flat in x when $\alpha = \overline{\alpha}_n$. Thus, the unique solution to the executive's optimization problem must be $x = \overline{x}_m(\beta)$.

Proof of (d): As before, $\beta > \frac{C}{B^L} + \frac{k}{2(1-q)B^L} \Leftrightarrow -(1-q) < \overline{x}_m(\beta) < 1-q$. Because $\alpha \in \left(\overline{\alpha}_n, \frac{C}{B^L}\right)$, then it is not possible for $\alpha \leq \overline{\alpha}_m(b^H, x)$ since $\overline{\alpha}_m(b^H, x) < \overline{\alpha}_n$ for all x. Moreover, from (73), $\overline{x}_m(\alpha, b^H) < -(1-q)$, while from (71), $\overline{x}_m(\alpha, b^L) < 1-q$. Thus, we can write $EU_a^E(x)$ as

$$EU_a^E(x) = \begin{cases} \alpha(pB^H + (1-p)B^L) - C - k & x \leq \overline{x}_m(\beta),\ x \leq \overline{x}_m(\alpha, b^L) \\ p\left(\left(\frac{2q-1}{q-x}\right)(\alpha B^H - C) - \left(1 + \frac{(1-q-x)(\alpha B^H - C)}{(q-x)(C-\alpha B^L)}\right)k\right) & x \leq \overline{x}_m(\beta),\ x \geq \overline{x}_m(\alpha, b^L) \\ \alpha(pB^H + (1-p)B^L) - C - k & x > \overline{x}_m(\beta),\ x \leq \overline{x}_m(\alpha, b^L) \\ p(q+x)(\alpha B^H - C) + (1-p)(1-q+x)(\alpha B^L - C) - k & x > \overline{x}_m(\beta),\ x \geq \overline{x}_m(\alpha, b^L) \end{cases}.$$

Thus we have three relevant cases: (i) $\overline{x}_m(\alpha, b^L) < -(1-q) < \overline{x}_m(\beta) < 1-q$ (ii) $-(1-q) < \overline{x}_m(\alpha, b^L) < \overline{x}_m(\beta) < 1-q$; (iii) $-(1-q) < \overline{x}_m(\beta) < \overline{x}_m(\alpha, b^L) < 1-q$.

Case (i): $\overline{x}_m(\alpha, b^L) < -(1-q) < \overline{x}_m(\beta) < 1-q$. In this case

$$EU_a^E(x) = p\left(\left(\frac{2q-1}{q-x}\right)(\alpha B^H - C) - \left(1 + \frac{(1-q-x)(\alpha B^H - C)}{(q-x)\,(C-\alpha B^L)}\right)k\right),$$

which strictly increases up to $\overline{x}_m(\beta)$. It then jumps down and becomes $EU_a^E(x) = p(q+x)(\alpha B^H - C) + (1-p)(1-q+x)(\alpha B^L - C) - k$, which is an increasing function when $\alpha \in \left(\overline{\alpha}_n, \frac{C}{B^L}\right)$. Thus there is a unique optimum, either at $EU_a^E(\overline{x}_m(\beta)) = p\left(\left(\frac{2q-1}{q-\overline{x}_m(\beta)}\right)(\alpha B^H - C) - \left(1 + \frac{(1-q-\overline{x}_m(\beta))(\alpha B^H - C)}{(q-\overline{x}_m(\beta))(C-\alpha B^L)}\right)k\right)$ or $EU_a^E(1-q) = p(q+1-q)(\alpha B^H - C) + (1-p)(1-q+1-q)(\alpha B^L - C) - k = p(\alpha B^H - C) - (1-p)2(1-q)(C - \alpha B^L) - k$, which ever is bigger.

Case (ii): $-(1-q) < \overline{x}_m(\alpha, b^L) < \overline{x}_m(\beta) < 1-q$. In this circumstance, $EU_a^E(x)$ is initially flat at $\alpha(pB^H + (1-p)B^L) - C - k$ along $[-(1-q), \overline{x}_m(\alpha, b^L)]$. At $\overline{x}_m(\alpha, b^L)$, $EU_a^E(x)$ becomes $p\left(\left(\frac{2q-1}{q-x}\right)(\alpha B^H - C) - \left(1 + \frac{(1-q-x)(\alpha B^H - C)}{(q-x)(C-\alpha B^L)}\right)\right.$ and begins to increase. Once it reaches $\overline{x}_m(\beta)$, $EU_a^E(x)$ jumps down and becomes $EU_a^E(x) = p(q+x)(\alpha B^H - C) + (1-p)(1-q+x)(\alpha B^L - C) - k$, which is an increasing function when $\alpha \in \left(\overline{\alpha}_n, \frac{C}{B^L}\right)$. Thus there is a unique optimum, either at $EU_a^E(\overline{x}_m(\beta)) = p\left(\left(\frac{2q-1}{q-\overline{x}_m(\beta)}\right)(\alpha B^H - C) - \left(1 + \frac{(1-q-\overline{x}_m(\beta))(\alpha B^H - C)}{(q-\overline{x}_m(\beta))(C-\alpha B^L)}\right)k\right)$ or $EU_a^E(1-q) = p(\alpha B^H - C) - (1-p)2(1-q)(C - \alpha B^L) - k$, which ever is bigger.

Case (iii): $-(1-q) < \overline{x}_m(\beta) < \overline{x}_m(\alpha, b^L) \leq 1-q$. In this case, $EU_a^E(x)$ is flat at $\alpha(pB^H + (1-p)B^L) - C - k$ all along $[-(1-q), \overline{x}_m(\alpha, b^L)]$. Once it reaches $\overline{x}_m(\beta)$, it remains flat (since $\overline{x}_m(\beta) < \overline{x}_m(\alpha, b^L)$) until it reaches $\overline{x}_m(\alpha, b^L)$. At that point $EU_a^E(x)$ becomes $p(q+x)(\alpha B^H - C) + (1-p)(1-q+x)(\alpha B^L - C) - k$, which strictly increases in x until $x = 1-q$. Thus, in this case, unique solution to the executive's optimization problem must be $x = 1-q$. Notice that $\overline{x}_m(\alpha, b^L) \in (\overline{x}_m(\beta), 1-q)$ if and only if $\alpha \in \left(\overline{\alpha}_m(b^L, \overline{x}_m(\beta)), \frac{C}{B^L}\right)$.■

Proof of Corollary 1. The result follows from parts (c) and (d) of Proposition OA10, recognizing that $\beta > \beta^m = \frac{C}{B^L} + \frac{k}{(1-q)B^L}$ implies $\beta > \frac{C}{B^L} + \frac{k}{2(1-q)B^L}$.

Proof of Proposition OA11.

Because $E[\phi_f(\widetilde{b})|B^L](\alpha B^L - C) - k < 0$ (since $\alpha B^L - C < 0$), the executive prefers that a low-type regulator not propose, i.e., $\rho_f^*(B^L) = 0$. The executive can implement $\rho_f^*(B^L) = 0$ by replacing (46) with the constraint

$$\left[(1-q)\phi_f(b^H) + q\phi_f(b^L)\right]\left(\beta B^L - C\right) - k \leq 0.$$

We solve the executive's problem conditional on a fixed $\rho_f(B^H) \in (0,1]$ and return later and find its optimal value. We can restate the executive's problem as

$$\max_{\phi_f(b^H), \phi_f(b^H)} p\left(\rho_f(B^H)\left\{\left[q\phi_f(b^H) + (1-q)\phi_f(b^L)\right](\alpha B^H - C) - k\right\}\right),$$

$$\text{subject to: } \left[(1-q)\phi_f(b^H) + q\phi_f(b^L)\right]\left(\beta B^L - C\right) - k \leq 0 \tag{79}$$

$$\phi_f(b^H) \in [0,1], \phi_f(b^H) \in [0,1]. \tag{80}$$

Because $\alpha B^H - C > 0$, the executive's objective function is strictly increasing in $\phi_f(b^H)$ and $\phi_f(b^L)$, and ideally, ignoring (79), it would want to set $\phi_f(b^H) = \phi_f(b^L) = 1$. Notice that it can do this when $\beta \leq \frac{C+k}{B^L}$. In this case, (79) is satisfied when $\phi_f(b^H) = \phi_f(b^L) = 1$.

Continuing to hold $\rho_f(B^H) \in (0,1]$ fixed, now suppose $\beta > \frac{C+k}{B^L}$. In this case, $\phi_f(b^H) = \phi_f(b^L) = 1$ is no longer feasible—(79) is violated. Thus, the executive must reduce $\phi_f(b^H)$ below 1 or $\phi_f(b^L)$ below 1 or both. Let us first argue that we cannot have $\phi_f(b^H) \in (0,1)$ and $\phi_f(b^L) \in (0,1)$. To see why, form the Lagrangian for the executive's problem and differentiate with respect to $\phi_f(b^H)$ and $\phi_f(b^L)$:

$$\begin{aligned}\mathcal{L} = {} & p\left(\rho_f(B^H)\left\{\left[q\phi_f(b^H) + (1-q)\phi_f(b^L)\right](\alpha B^H - C) - k\right\}\right) \\ & - \lambda\left\{\left[(1-q)\phi_f(b^H) + q\phi_f(b^L)\right]\left(\beta B^L - C\right) - k\right\} \\ & - \xi_{1H}(\phi_f(b^H) - 1) + \xi_{0H}\phi_f(b^H) - \xi_{1L}(\phi_f(b^L) - 1) + \xi_{0L}\phi_f(b^L),\end{aligned}$$

where λ is the multiplier for (79) and $\xi_{1H}, \xi_{0H}, \xi_{1L}, \xi_{0L}$ are the multipliers for the constraints in (80). If these latter constraints are all slack then

$$\frac{\partial \mathcal{L}}{\partial \phi_f(b^H)} = 0 \Rightarrow p\rho_f(B^H)q(\alpha B^H - C) = \lambda(1-q)\left(\beta B^L - C\right)$$
$$\frac{\partial \mathcal{L}}{\partial \phi_f(b^L)} = 0 \Rightarrow p\rho_f(B^H)(1-q)(\alpha B^H - C) = \lambda q\left(\beta B^L - C\right),$$

which in turn implies $\lambda = \frac{p\rho_f(B^H)q(\alpha B^H - C)}{(1-q)(\beta B^L - C)} = \frac{p\rho_f(B^H)(1-q)(\alpha B^H - C)}{q(\beta B^L - C)}$. But this cannot hold because $\frac{q}{1-q} > \frac{1-q}{q}$. By similar reasoning, we can rule out the possibility that $\phi_f(b^H) = 0$ and $\phi_f(b^L) \in (0,1]$. It is also straightforward to rule out $\phi_f(b^H) = 0$ and $\phi_f(b^L) = 0$. Thus, when $\beta > \frac{C+k}{B^L}$, we have three possibilities for the executive's optimum: (i) set $\phi_f(b^H) = 1$ and reduce $\phi_f(b^L)$ below 1; (ii) set $\phi_f(b^H) < 1$, while reducing $\phi_f(b^L)$ to 0; (iii) set $\phi_f(b^H) < 1$ while keeping $\phi_f(b^L) = 1$. We now argue that possibility (iii) cannot be optimal, while possibilities (i) and (ii) constitute the optimal solution, depending on parameter values.

Let us now argue that we will reduce $\phi_f(b^L)$ as much as we can while holding $\phi_f(b^H) = 1$. Suppose we reduce $\phi_f(b^L)$ by Δ_L (from 1 to $1 - \Delta_L$) by the smallest amount we can consistent with incentive compatibility. That is, we would chose Δ_L so that (79) just holds with $\phi_f(b^H) = 1$:

$$\left[(1-q) + q(1-\Delta_L)\right]\left(\beta B^L - C\right) = k,$$

or

$$\Delta_L = 1 - \frac{k}{q\left(\beta B^L - C\right)} + \frac{(1-q)}{q}.$$

Now, suppose instead, we hold $\phi_f(b^L) = 1$ and reduce $\phi_f(b^H)$ by Δ_H (from 1 to $1 - \Delta_H$) so that (79) just holds:

$$\left[(1-q)(1-\Delta_H) + q\right]\left(\beta B^L - C\right) = k,$$

or

$$\Delta_H = 1 - \frac{k}{(1-q)\left(\beta B^L - C\right)} + \frac{q}{1-q}.$$

Taking the difference

$$\begin{aligned}\Delta_H - \Delta_L &= \left(1 - \frac{k}{(1-q)\left(\beta B^L - C\right)} + \frac{q}{1-q}\right) - \left(1 - \frac{k}{q\left(\beta B^L - C\right)} + \frac{(1-q)}{q}\right) \\ &= \frac{2q-1}{q(1-q)}\left(1 - \frac{k}{\left(\beta B^L - C\right)}\right) > 0,\end{aligned}$$

because $q > \frac{1}{2}$ and we are in the range where $\beta > \frac{C+k}{B^L}$. Now, with perturbation Δ_L, the objective function then goes down by

$$\Delta EU_L^E = p\rho_f(B^H)(1-q)(\alpha B^H - C)\Delta_L, \tag{81}$$

while with perturbation Δ_H, the objective function goes down by

$$\Delta EU_H^E = p\rho_f(B^H)q(\alpha B^H - C)\Delta_H.$$

Because $\Delta_H > \Delta_L$ and $q > 1-q$, the objective function goes down more by perturbing $\phi_f(b^H)$ downward than by perturbing $\phi_f(b^L)$ downward. Thus, the best thing for the executive to do is to hold $\phi_f(b^H) = 1$ and reduce $\phi_f(b^L)$ by just enough so that (79) holds as an equality, i.e.,

$$\phi_f(b^L) = \frac{k}{q\left(\beta B^L - C\right)} - \frac{(1-q)}{q}.$$

Now, this will be feasible if and only if $\phi_f(b^L) \geq 0$, or

$$\beta \leq \overline{\beta}_m = \frac{k}{(1-q)\,B^L} + \frac{C}{B^L}.$$

Thus, when $\beta \in \left(\frac{C+k}{B^L}, \overline{\beta}_m\right)$, the executive's optimal solution is

$$\begin{aligned}\phi_f(b^H) &= 1,\\ \phi_f(b^L) &= \frac{k}{q\,(\beta B^L - C)} - \frac{(1-q)}{q}.\end{aligned}$$

Now, when $\beta > \overline{\beta}_m$, we will have reduced $\phi_f(b^L)$ all the way to zero, and the only way to maintain incentive compatibility is to reduce $\phi_f(b^H)$ so that (79) holds as an equality, i.e.,

$$\phi_f(b^H) = \frac{k}{(1-q)\,(\beta B^L - C)}.$$

Summarizing, holding $\rho_f(B^H) \in (0,1]$ fixed, the solution to the executive's problem is

$$\phi_f^*(b^H) = \begin{cases} 1 & \beta \leq \frac{C+k}{B^L} \\ 1 & \beta \in \left(\frac{C+k}{B^L}, \overline{\beta}_m\right) \\ \frac{k}{(1-q)(\beta B^L - C)} & \beta \geq \overline{\beta}_m \end{cases}. \tag{82}$$

$$\phi_f^*(b^L) = \begin{cases} 1 & \beta \leq \frac{C+k}{B^L} \\ \frac{k}{q(\beta B^L - C)} - \frac{(1-q)}{q} & \beta \in \left(\frac{C+k}{B^L}, \overline{\beta}_m\right) \\ 0 & \beta \geq \overline{\beta}_m \end{cases}. \tag{83}$$

Therefore

$$E\left[\phi_f^*(\widetilde{b})|B^H)\right] = \begin{cases} 1 & \beta \leq \frac{C+k}{B^L}. \\ q + (1-q)\left[\frac{k}{q(\beta B^L - C)} - \frac{(1-q)}{q}\right] & \beta \in \left(\frac{C+k}{B^L}, \overline{\beta}_m\right). \\ q\frac{k}{(1-q)(\beta B^L - C)} & \beta \geq \overline{\beta}_m. \end{cases} \tag{84}$$

The executive's maximized welfare, conditional on $\rho_f(B^H) \in (0,1]$, is

$$EU_f^E(\rho_f(B^H)) = p\rho_f(B^H)\left\{E\left[\phi_f^*(\widetilde{b})|B^H)\right](\alpha B^H - C) - k\right\}. \tag{85}$$

We can use this to determine the optimal value of $\rho_f(B^H)$. Note that a value of $\rho_f(B^H) = 0$ can be implemented by setting $\phi_f^*(b^H) = \phi_f^*(b^L) = 0$. Also note that a value of $\rho_f(B^H) = 1$ can be implemented by using the approval probabilities in (82) and (83). This is because these solutions satisfy (79) as an equality, which in turn implies that the approval probabilities in (82) and (83) are such that $E[\phi_f^*(\widetilde{b})|B^H](\beta B^H - C) - k > 0$. From (85), we then have.

$$\rho_f^*(B^H) = \begin{cases} 1 & \alpha \geq \frac{C}{B^H} + \frac{k}{E[\phi_f^*(\widetilde{b})|B^H]B^H} \\ 0 & \text{otherwise} \end{cases}.$$

■

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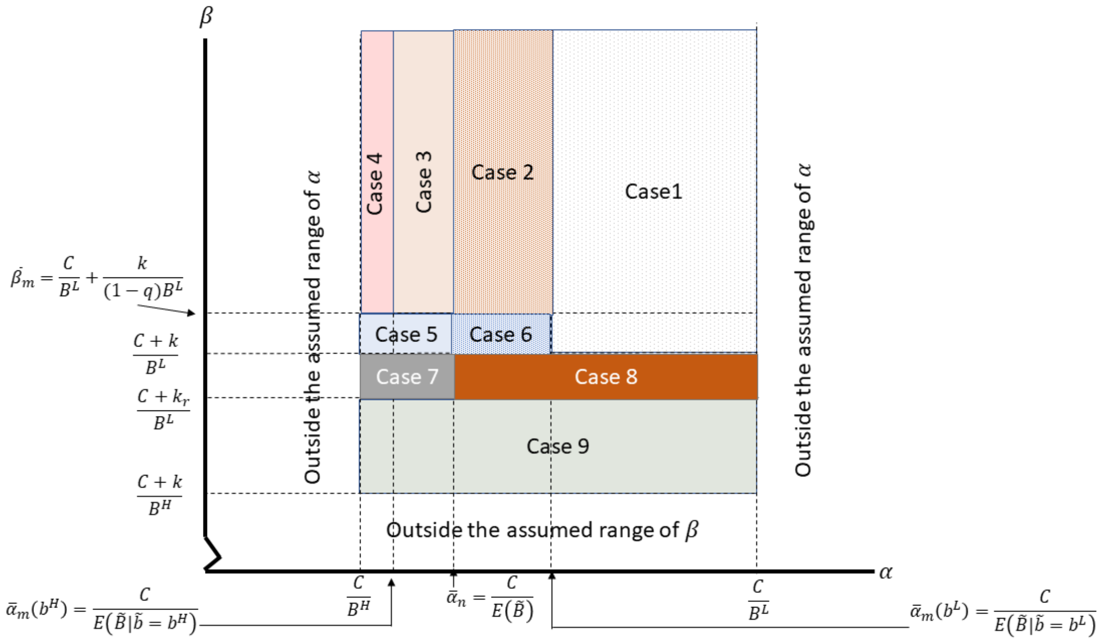


Figure OA1: When $\beta > \frac{C+k}{B^H}$ and $k_a > 0$, there are nine cases for $\Delta^E(\alpha, \beta)$.

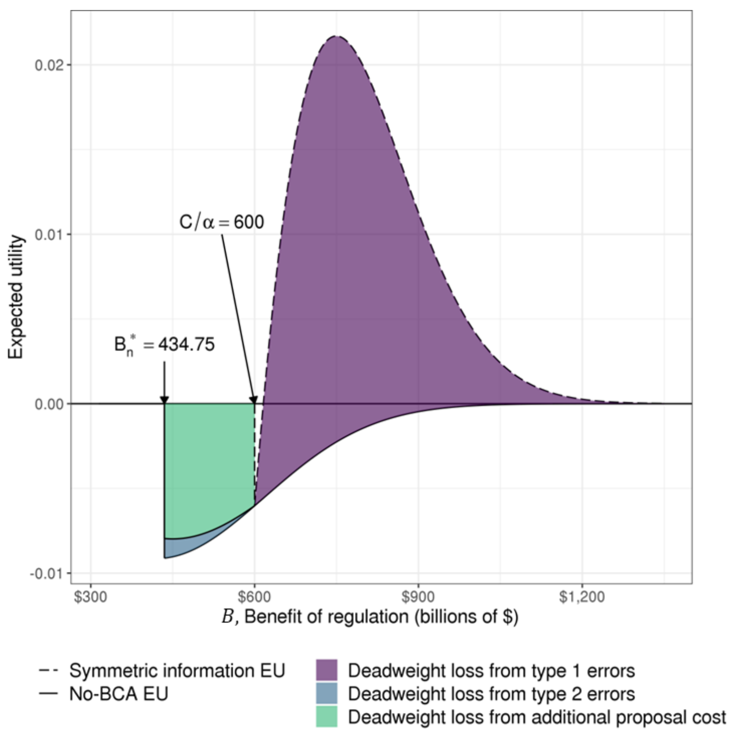


Figure OA2: The three components of deadweight loss to the executive when it does not use BCA: losses from type 1 and type 2 errors and the loss from additional proposal costs. The figure depicts the case in which $\alpha = 0.25$, with all other parameters set at baseline levels.

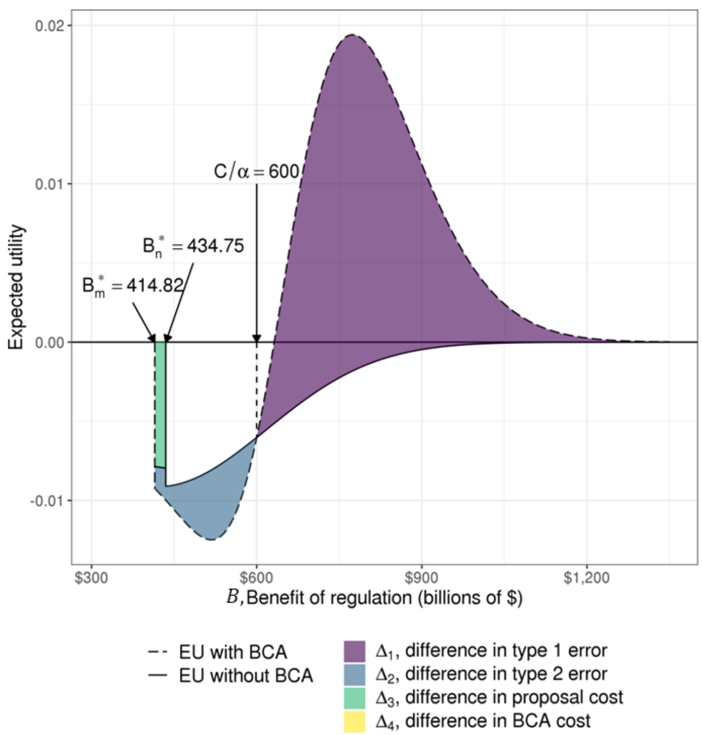


Figure OA3: The decomposition terms Δ_1, Δ_2, Δ_3, and Δ_4 for the change in the executive's welfare due to the use of BCA. The figure illustrates the case in which $\alpha = 0.25$ and all other parameters are at their baseline values.

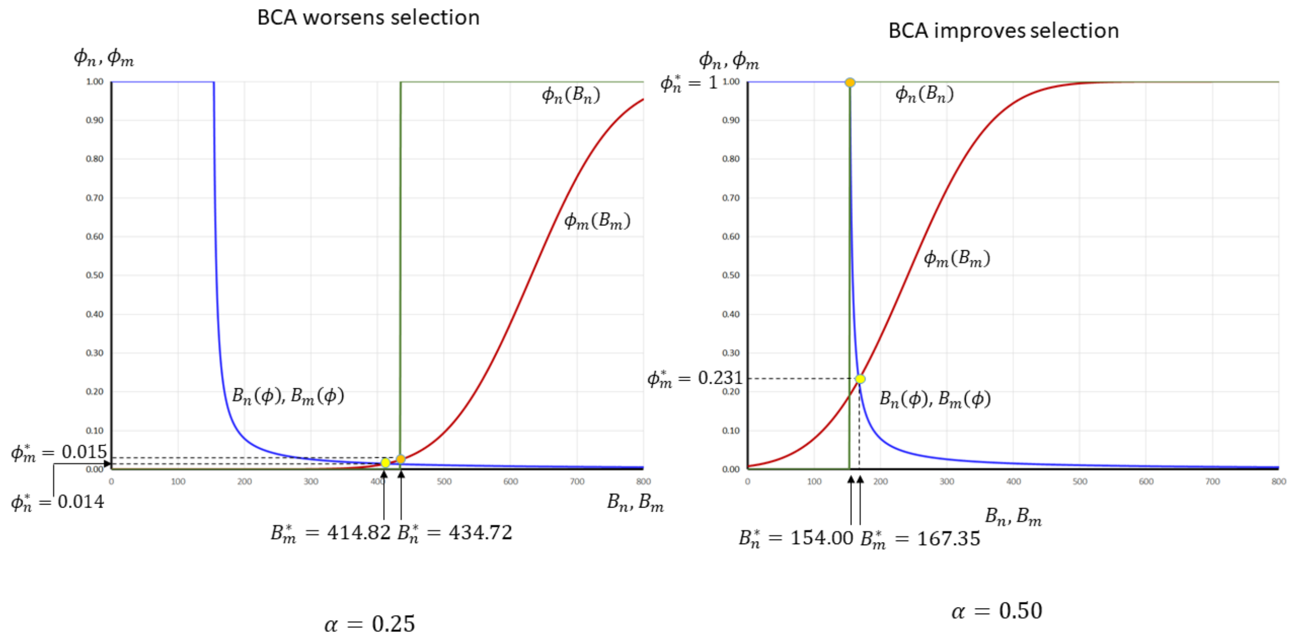


Figure OA4: The right-hand panel illustrates the equilibria with and without BCA for the baseline parameterization ($\alpha = 0.50$ and all other parameters are set to their baseline levels.) It illustrates a case in which BCA improves selection. The left-hand panel illustrates the equilibria when $\alpha = 0.25$ and all other parameters set to their baseline levels. It illustrates a case in whch BCA worsens selection.

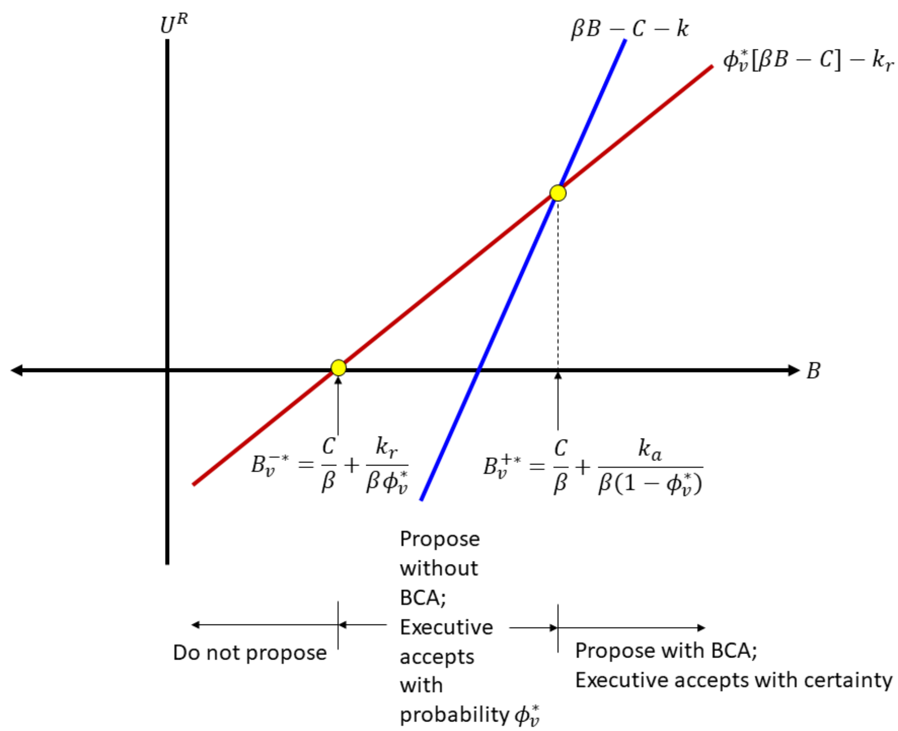


Figure OA5: Separating equilibrium under voluntary BCA.

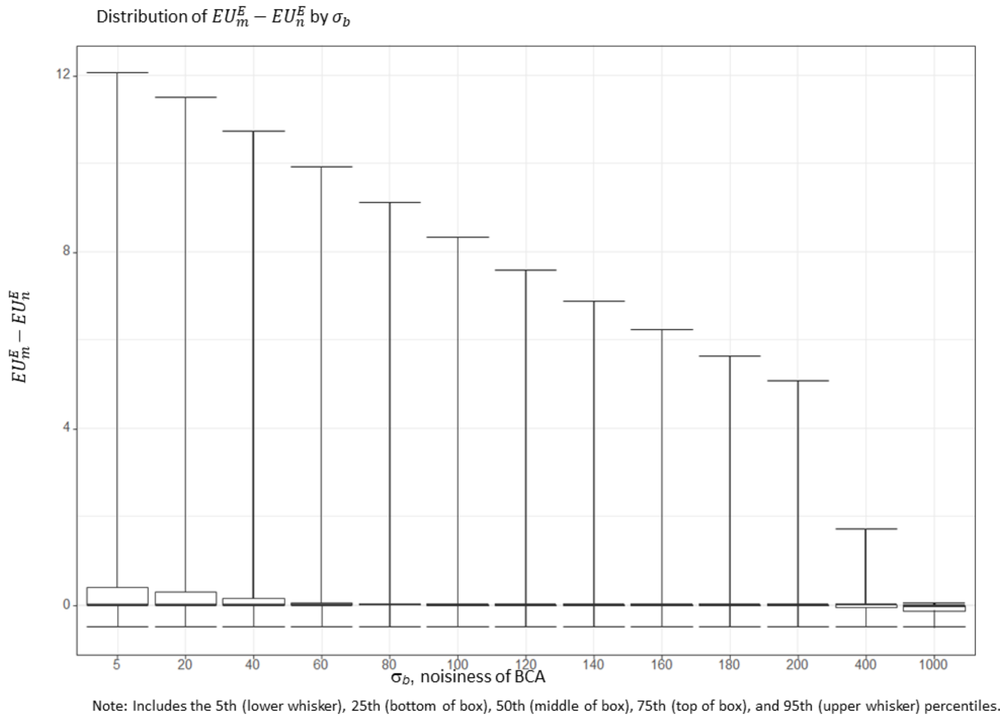


Figure OA6: Distribution of $EU_m^E - EU_n^E$ across all parameterizations for various values of the noisiness of BCA, σ_b.

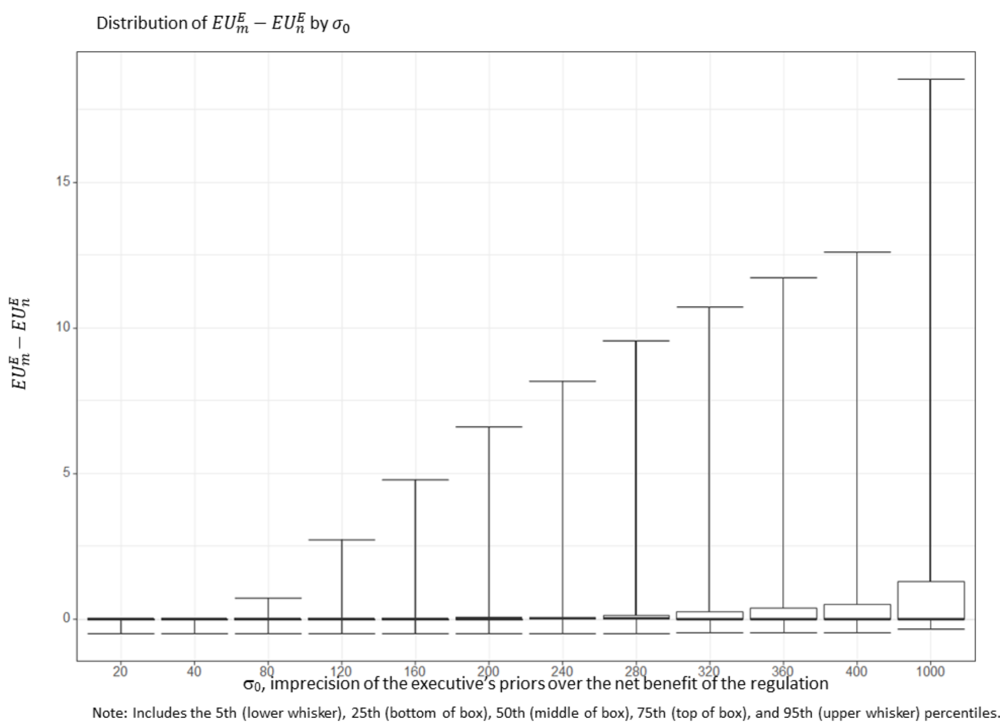
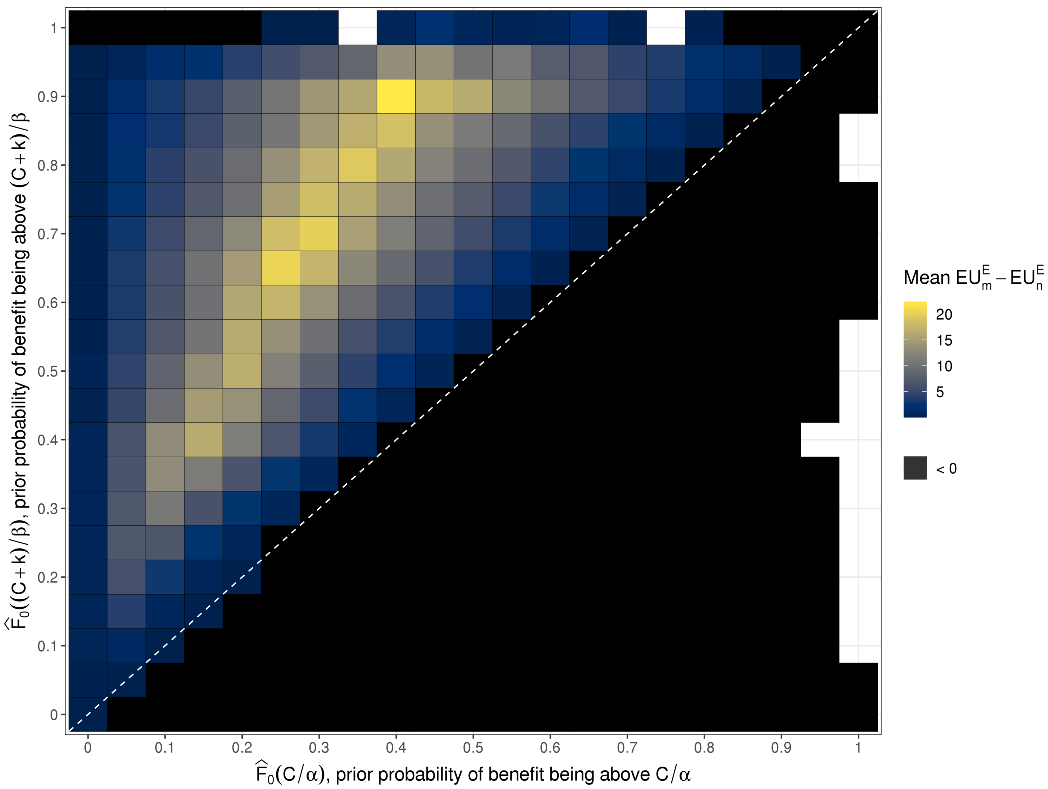
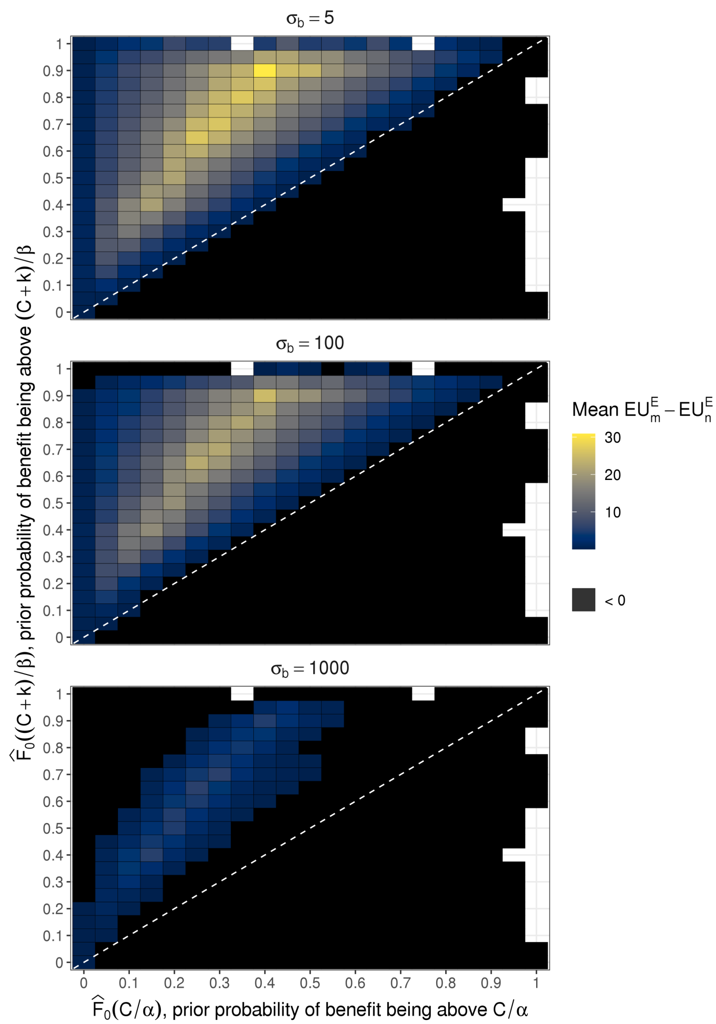


Figure OA7: Distribution of $EU_m^E - EU_n^E$ across all parameterizations for various values of the imprecision of the executive's priors, σ_0.



Note: Blank spaces refer to values that do not exist in our parameter space.

Figure OA8: $EU_m^E - EU_n^E$ for various values of the regulation-aversion indices, $\widehat{F}_0\left(\frac{C}{\alpha}\right)$ and $\widehat{F}_0\left(\frac{C+k}{\beta}\right)$, of the executive and regulator. All parameters vary over their entire ranges.



Note: Blank spaces refer to values that do not exist in our parameter space.

Figure OA9: $EU_m^E - EU_n^E$ for various values of the regulation-aversion indices, $\widehat{F}_0\left(\frac{C}{\alpha}\right)$ and $\widehat{F}_0\left(\frac{C+k}{\beta}\right)$, for $\sigma_b = 5, 100$, and $1,000$. All other parameters vary over their entire ranges.

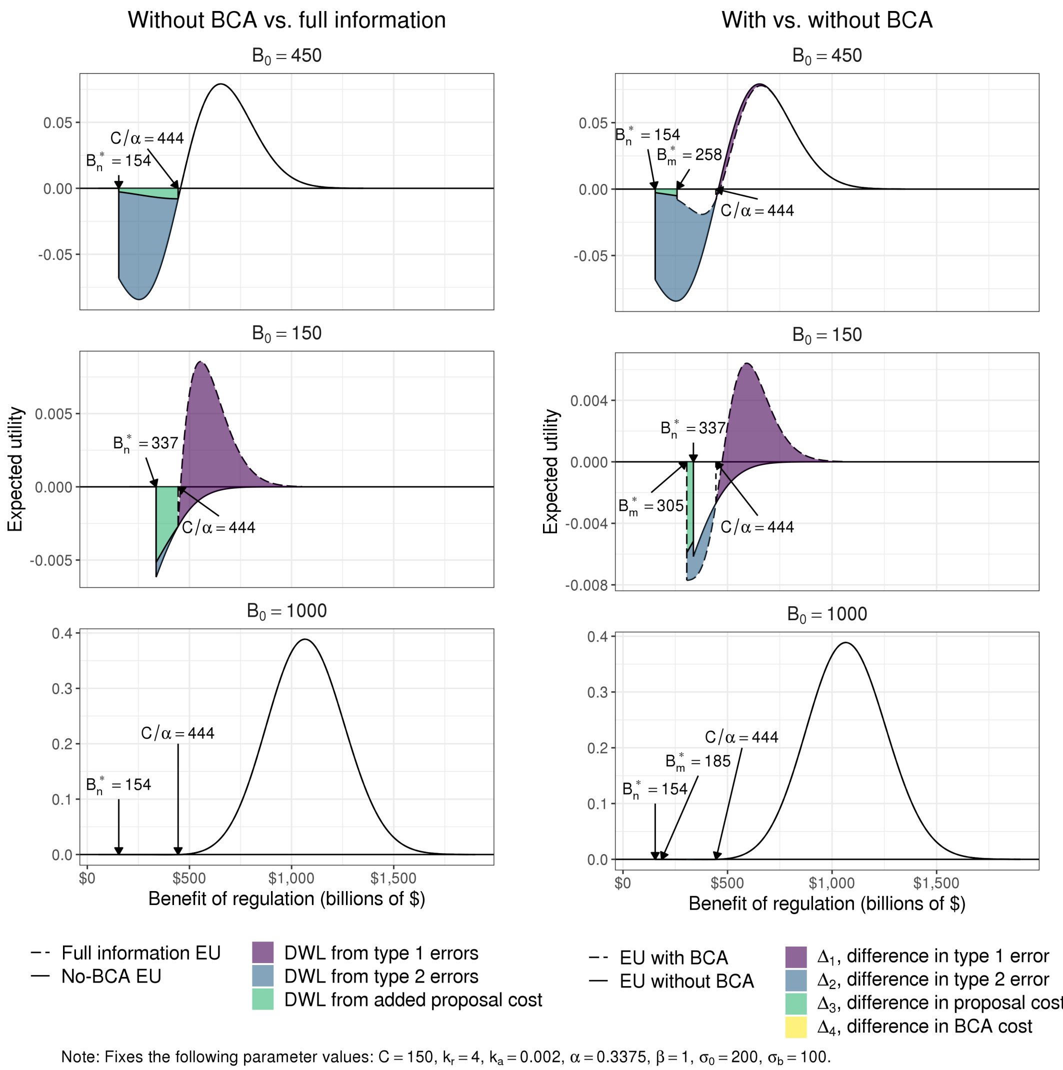


Figure OA10: Welfare decomposition for $\alpha = 0.3375$, $B_0 = \$450, \$150, \$1,000$ and all other parameter values at baseline levels.

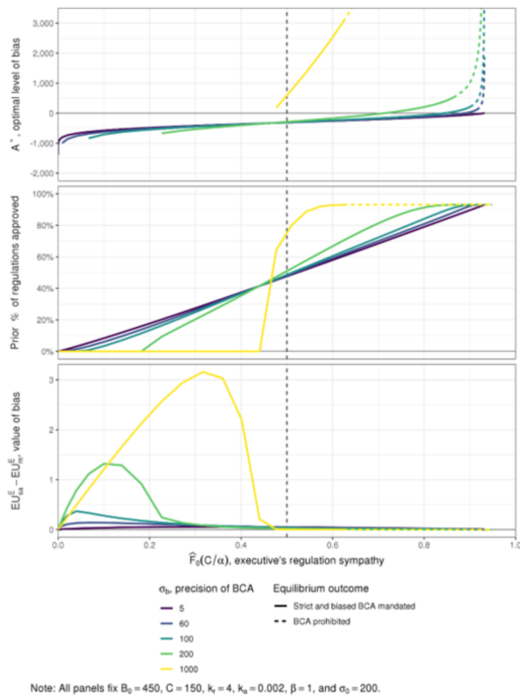


Figure OA11: The top panel shows the executive's optimal bias A^* as a function of the executive's regulation-sympathy index. (This constitutes the horizontal axis for the other two panels, too.) The middle panel plots the executive's prior probability of approving a regulation using strict BCA with bias, $\int_{B^*_{sa}}^{\infty} \widehat{\Psi}\left(\frac{C-A^*-x}{\sigma_b}\right) f_0(x)dx$. The bottom panel shows the executive's gain from bias, $EU^E_{sa} - EU^E_m$, relative to a discretionary BCA mandate.